CSO OPTICS MEMO #1 **January 3, 1992**

TO: All
FROM: Gene Serabyn
SUBJECT: **CSO Chopping Secondary Motion Requirements**

The new chopping secondary mirror for the CSO will need to be accurately positioned, both along the optical axis (z coordinate), and perpendicular to it (x,y coordinates). In addition, it will need to be correctly tilted so that, in its centered position, its optical axis lines up with the optical axis of the primary. Non zero errors in these 3 translational and 2 rotational degrees of freedom will introduce focus, decenter and tip-tilt errors into the image. The following discussion relates the sizes of these errors to the resultant image quality, for the wavelength range appropriate to the CSO,

$$0.3 < \lambda < 1.5\,\mathrm{mm}.$$

The analysis uses theoretical formulae whenever possible, and also calculations from the wavefront analysis section of the optics program Code V. A convenient diffraction-based measure of the image quality is the Strehl ratio, S, which for small wavefront aberrations is related to the rms wavefront error in the exit pupil, σ, via

$$S = \exp - \left(\frac{2\pi\sigma}{\lambda}\right)^2.$$

This functional form is valid only for $S \gtrsim 0.6$ or so. For reference, this function is plotted in Fig. 1. Since S increases with λ, the following calculations of the Strehl ratio will generally be carried out for the 'worst case scenario' provided by the shortest wavelength.

For convenience, numerical values of the telescope optical parameters are given in Fig. 2. In the following discussion, small f's will be used to refer to focal lengths, capital F's to focal ratios (F#'s), D's to mirror diameters, R's and ρ's to mirror and focal plane radii of curvatures, respectively, θ_{sec} to the rotation angle of the secondary mirror, and θ_{sky} to the angular displacement of the beam on the sky.

1. Ideal (collimated) optics

The plate scale, P, in the Cassegrain focal plane is determined by the focal length of the single-mirror "equivalent paraboloid" via $y_{cass} = f_{eq}\theta_{sky}$. Since $f_{eq} = m f_{pri}$, where m, the transverse magnification of the secondary ($= \frac{c+a}{c-a}$; Fig. 2), is 30.285, this gives $f_{eq} = 124.87$ m, and $P = 1.65''/\mathrm{mm}$.

The focal surface is, however, not flat, but part of a sphere which is open toward the secondary. Because of off-axis astigmatism, the tangential and sagittal rays come to a focus on two slightly different surfaces, the median of which has a radius of curvature[1]:

$$\rho_{med} = \frac{R_{pri}}{2}\left(\frac{m^2(1+\beta)}{(m^2-2)(m-\beta)+m(m+1)}\right).$$

where β is the distance that the Cassegrain focal plane lies behind the primary vertex, in units of f_{pri}. Inserting $\beta = 0.77408$, we get $\rho_{med} = 239.9\,\mathrm{mm}$, a rather small number. Note also that $\rho_{med} \approx \frac{1}{2}R_{sec}$. The focal plane curvature determines the z displacement to best off-axis focus through

$$\Delta z_{cass} \approx \frac{y_{cass}^2}{2\rho_{med}} = \frac{\theta_{sky}^2}{2\,P^2\,\rho_{med}}$$

$$\text{or} \quad \Delta z_{cass} = 2.75\,\text{mm}\ \theta^2_{sky}\,(\text{arc min}).$$

Code V gives the actual numerical factor as roughly 2.7. The resultant off-axis focal surface is plotted in Fig. 3. Because of this curvature of field, the Strehl ratio for an off-axis point in the paraxial (flat) Cassegrain focal plane drops to 0.9 approximately 5′ off axis, for $\lambda = 300\ \mu$m. However, along the curved 'best focal surface', the Strehl ratio is close to 1 out to at least 10 arc min from the axis.

2. Secondary Translation

2.1. Focus (z) motion:

If the secondary is translated along the z-axis, the focal spot translates along the z-axis in the same direction (for a Cassegrain telescope). In the paraxial approximation, the ratio of this image z motion, Δz_{cass}, to the secondary translation, Δz_{sec}, is given by $m^2 + 1 = 919$. Although verified by Code V for the paraxial approximation, this is not the full answer, since a z displacement of the secondary introduces spherical aberration in addition to defocus. Thus, the 'best' focus, in terms of Strehl ratio, does not move quite this far, but, according to Code V, only about 3/4 of this distance. Numerically, Code V yields

$$\frac{\Delta z_{cass}}{\Delta z_{sec}} \approx 670.$$

The necessary range of z-travel for the secondary is then determined by the amount of z motion necessary to generate a focus curve at the longest wavelength considered, $\lambda = 1.5$mm. Near focus, the axial distribution of intensity can be described by a sinc function[2],

$$I(z) = I(0)\ \text{sinc}^2\left(\frac{kz}{16F^2}\right) = I(0)\ \frac{\sin(\frac{kz}{16F^2})}{\frac{kz}{16F^2}},$$

where $k = \frac{2\pi}{\lambda}$, and z is the axial distance from the Cassegrain focal plane. This function has first zeros at

$$\frac{kz}{16F^2} = \pm\pi,$$

$$\text{or} \quad z = \pm 8F^2\lambda \approx \pm 1200\lambda.$$

In terms of secondary motion, this corresponds to $\Delta z_{sec} = 1.8\lambda$. Since we don't really have to defocus all the way to zero, a motion of $\pm\lambda$ likely will suffice to get to the half-power points (verified by Code V). Nevertheless, to be conservative, we take the required range as $\pm 1.5\lambda$. For a maximum λ of 1.5 mm this implies a motion of 4.5 mm. Adding another 3 mm for shifting the center of the range with elevation, and 2.5 mm for offsetting the overall range to compensate for mechanical tolerances, a maximum secondary travel of about 10 mm is required.

To determine the necessary resolution, consider an 0.5% drop from the maximum intensity, so that

$$0.995 = \frac{\sin u}{u} \approx \frac{1}{u}(u - \frac{u^3}{3!}).$$

The solution to this equation is $u = 0.17$, which implies

$$z_{res} = 0.17\left(\frac{8F^2\lambda_{min}}{\pi}\right) \approx 0.1\lambda_{min}.$$

For $\lambda_{min} = 300\,\mu m$, this implies a minimum step size of $30\,\mu m$. A focus curve can also be calculated with the aid of Code V (Fig. 4). Using the same 0.5% criterion then yields the slightly different result of $22\,\mu m$, which again is related to the effect of spherical aberration on the Code V calculation.

The required z motion is then: Range = 10 mm; Step size = $20\,\mu m$; Dynamic range = 500.

2.2. Secondary Decenter

The solid lines in Fig. 5 show the Strehl ratio vs. secondary decenter calculated with Code V for the minimum and maximum wavelengths considered. As the figure shows, for any wavelength, a decenter of only $\frac{4}{3}\lambda$ causes the Strehl ratio to drop to about 0.9, and a decenter of 2λ implies $S = 0.8$. By translating the secondary through its best lateral position, a 'decenter' curve can be acquired, similar to a focus curve. As evident in the figure, the travel required to get to the half power points is about $\pm 3\lambda$. To this must be added the travel needed to compensate for the sagging of the feed legs with elevation, about 3 mm. Considering the longest wavelength then, a travel of roughly 12 mm is required.

If the resolution is determined by the same level of controllability (0.5%) as the last case, the shortest wavelength requires a minimum decenter step size of 0.1 mm (Fig. 5).

The required x, y motions are then: Range = 12 mm; Resolution = $100\,\mu m$; Dynamic Range = 120.

3. Secondary Rotation

First consider a symmetric rotation (chop) about the prime focus of an otherwise perfectly aligned secondary. For large enough chop angles, the illumination pattern will begin to spill over the primary. Using simple geometric optics, this occurs for $\theta_{sky} = \pm 4.3'$.

Chopping has the effect of laterally translating the curved image surface (Sec. 1) across the optical axis, introducing focus shifts dependant on chop angle. In line with naive expectations, Code V verifies that this focus shift is given roughly by the off-axis defocus curve of Fig. 3, with the angle on the sky now being the chop angle. As can be seen from this figure, for a $\pm 3'$ chop about the optical axis, the best focus moves toward the secondary by about 2.5 cm. At this best focus, the Strehl ratio is 0.967, corresponding to $\sigma = 0.029\lambda$, while at the nominal focus, $S = 0.953$ and $\sigma = 0.035\lambda$. Since this effect is small, even for this 'worst' case (short wavelength and large chop angle), refocusing to correct for this effect will not be required.

For the case of a single on-axis detector, the dependance of the Strehl ratio on the size of the chop angle is shown by the solid curve in Fig. 6. In order to keep the Strehl ratio greater than 0.9 (0.95) at $\lambda = 300\mu m$, θ_{sky} must be less than $4'$ ($3'$). For longer wavelengths and the same chop angle, the Strehl ratio is even higher. Since these limits are comparable to both the mechanical limit of the chopper, and to the spillover limit discussed above, a general purpose limit on θ_{sky} of ± 4 to ± 5 arc min is implied. The accuracy of the chop angle will need to be about $0.5''$ on the sky. Using the relationship between secondary rotation and the resultant angular displacement of the beam on the sky[3] (Appendix),

$$\theta_{sky} = 2\frac{(c-a)}{f_{pri}}(1 - \frac{c-a}{R_{sec}})\theta_{sec},$$

we get $\theta_{sec} = 17.07705\theta_{sky}$. Thus we have the following requirements for the secondary chop angle sensor:

Range = $2.85°$; Resolution = $8.5''$; Dynamic range = 1200.

A multi-element detector array brings with it one additional complication: since the focal plane is curved, the section of the off-axis image surface that is translated onto the axis by the chopper mirror has a non-zero average slope. In other words, the resultant best focal plane is tilted. This means that the best focus for points on opposite sides of the nominal chop angle will be on opposites side of the array plane (closer to and further away from the secondary, for points beyond and inside the nominal chop, respectively). Since the focal plane tilt is in opposite directions for two symmetric chop positions, this problem cannot be corrected later down the optical train. Refocusing is also of no use, since some pixels get worse while others get better.

The main result of the tilted 'best' focal plane, for a fixed array plane, is that the Strehl ratio at the edges of the array will differ from the central value. To see the size of this effect, a rectangular array of size $10 \times 1.22\lambda/D_{pri}$ (in terms of angular field of view) was considered. At $\lambda = 300\ \mu m$, the array size is $1.2'$. The range of Strehl ratios found across such an array is indicated in Fig. 6 by the two dotted curves. The upper curve shows the Strehl ratio along the 'inner' edge of the array (inside the chop), while the lower curve shows S along the outer edge of the array. The net effect of the array size at λ_{min} is thus to give a range of about $\pm 10\%$ to S, not a terribly large effect. Since S approaches 1 for longer λ's, the range narrows.

A second effect of the tilted focal plane might be thought to be a change in plate scale in the directions parallel and perpendicular to the chop direction, but this effect was found to be below the 1% level with Code V.

4. Simultaneous secondary rotation and decenter

or, can the chop degree of freedom be used to compensate for decenter errors ?

Assume the secondary is decentered by y_{dec}, and consider the y-z plane (Fig. 7). An incoming ray in this plane which is parallel to the telescope axis, but offset from it so that after reflection at the primary it hits the secondary upon its offset center, makes an angle

$$\theta_{dec} = \frac{y_{dec}}{c - a}$$

with the optical axis. This means that the axial image is shifted laterally by

$$\Delta y_{cass} = \theta_{dec}((c + a) - (c - a)) = 2a\theta_{dec}.$$

To recenter the image, the secondary must be tipped by θ_{sec}, given by (Appendix)

$$\Delta y_{cass} = 2(a + c)(1 - \frac{c - a}{R_{sec}})\theta_{sec}.$$

A glance at Fig. 7 indicates that the rotation direction needed is that which moves the center of the secondary even further off-axis, an undesirable state of affairs. Nevertheless, setting these two expressions for Δy_{cass} equal to each other yields

$$\theta_{sec} = \theta_{dec} \frac{1}{1 + \frac{c}{a}} \frac{1}{1 - \frac{c-a}{R_{sec}}}.$$

Since the eccentricity of the secondary is given by $\epsilon = c/a$ and since $R_{sec} = a(\epsilon^2 - 1)$, this works out to

$$\theta_{sec} = \frac{\theta_{dec}}{\epsilon}.$$

This angle adds to θ_{dec}, so that the secondary vertex-prime focus line now makes an even greater angle of $\theta_{dec}(1 + \frac{1}{\epsilon})$ with the optical axis.

Finally, using again the relationship between θ_{sky} and θ_{sec} (Appendix),

$$\theta_{sky} = 2\frac{(c-a)}{f_{pri}}(1 - \frac{c-a}{R_{sec}})\theta_{sec},$$

in the next to last equation, we get

$$\theta_{sky} = \frac{2y_{dec}}{f_{pri}(\epsilon + 1)}.$$

Numerically this gives

$$\theta_{sky}\,[\mathrm{arc\,sec}] = 48.4\, y_{dec}\,[\mathrm{mm}].$$

This means that to remove a 1 mm decenter error, the beam on the sky would need to move off-axis by 48.4″, a rather significant conversion ratio. A quick run of Code V verified this conversion factor.

However, the main problem is that tipping the secondary does not undo the optical effect of decentering the mirror, as was verified by Code V. Naively, the image quality should be degraded both by the chopping angle and by the decenter. However, since rotations about the prime focus do not degrade the image quality much (Fig. 6), the decenter should provide the dominant effect. In a sample run with Code V at a wavelength of 300μm, the secondary was first rotated so that a source located off-axis by 20″ was imaged at the center of the focal plane. Because of the small chop angle, this yielded almost no degradation of the Strehl ratio. Next, the secondary was decentered by 0.4133 mm, as per the above formula, in order to instead image an on-axis source onto the center of the focal plane. This final image had a Strehl ratio of roughly 0.9, which is exactly the same as the case of the image for a decentered, but non-tilted secondary considered in Fig. 5. Thus, inverting the steps, by now decentering first, and then rotating the secondary, it is evident that tilting the secondary has no effect on image aberrations due to the secondary decenter error. To first order, tipping the secondary only shifts the given point spread function in the focal plane. (Of course, for very large chop angles, tipping does degrade the image). Thus, it is not possible to correct secondary decenter errors with the chop angle degree of freedom.

Alternatively, starting again with a decentered secondary, one can instead *require* that the tipping of the secondary mirror be such that we move the mirror vertex back onto the axis, even though the back focal point of the secondary (which should be coincident with the prime focus) remains decentered. This then incurs a fixed pointing offset for the telescope. Nevertheless, how does the image quality fare? As a first guess, badly, because this situation is equivalent to chopping about the mirror vertex, a case known to possess poor image quality. Code V indeed verified this conclusion, as the same Strehl ratios were obtained for this case as for the case of a simple decenter of the same magnitude as the resultant back focus decenter. Thus, this also is not the way to go, and the conclusion is that the back focus of the secondary needs to be very accurately located at the prime focus, regardless of what else happens.

5. Summary of motion requirements

Focus (z) translation: Range = 10 mm; Step size = 20 μm; Dynamic range = 500.
Decenter (x,y) translations: Range = 12 mm; Resolution = 100 μm; Dynamic Range = 120.
Secondary rotation about chop axis: Range = 2.85°; Resolution = 8.5″; Dynamic range = 1200.

Appendix: Relation of secondary rotation angle to beam displacement in the paraxial approximation

Upon rotation through angle θ_{sec} of the secondary about a point behind the mirror vertex by distance δ, the slope of the region of the mirror now situated on the optical axis of the primary is

$$\theta_{axis} = \theta_{sec} - \frac{y}{R_{sec}},$$

where the last term is the original slope of this section of the mirror, which was a distance y off-axis before being tipped. Since $y = \delta\theta_{sec}$ (Fig. 8), we get

$$\theta_{axis} = \theta_{sec}\left(1 - \frac{\delta}{R_{sec}}\right).$$

The image is then displaced in the focal plane by

$$\Delta y_{cass} = 2\theta_{axis}(c+a) = 2(c+a)\left(1 - \frac{\delta}{R_{sec}}\right)\theta_{sec}. \qquad (1)$$

Since the plate scale is given by

$$\Delta y_{cass} = f_{pri}\theta_{sky}\left(\frac{c+a}{c-a}\right),$$

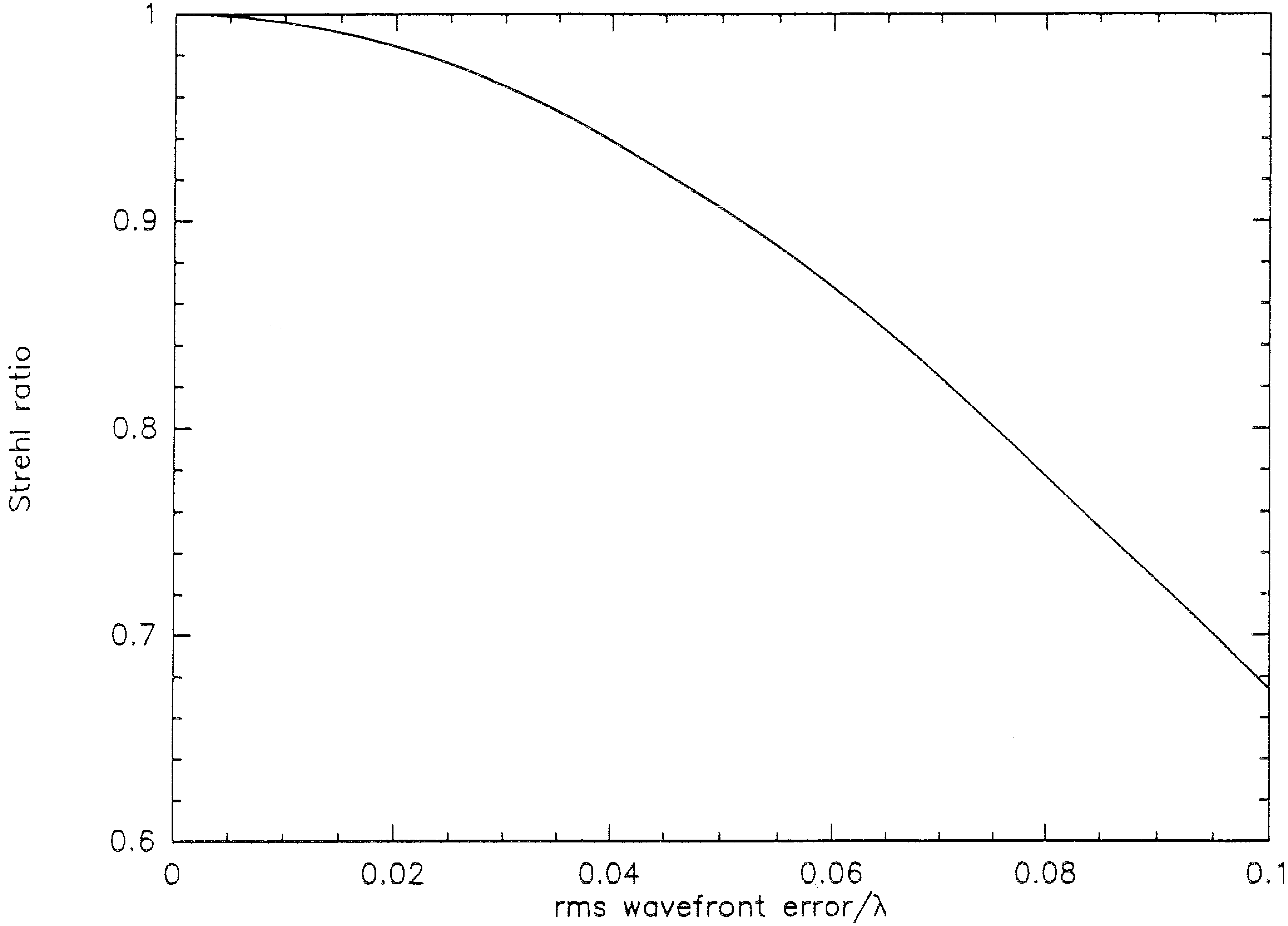
we get

$$\theta_{sky} = 2\left(\frac{c-a}{f_{pri}}\right)\left(1 - \frac{\delta}{R_{sec}}\right)\theta_{sec}. \qquad (2)$$

For rotation about the prime focus, as is assumed in the text, it remains only to insert $\delta = c - a$ in equations 1 and 2.

References

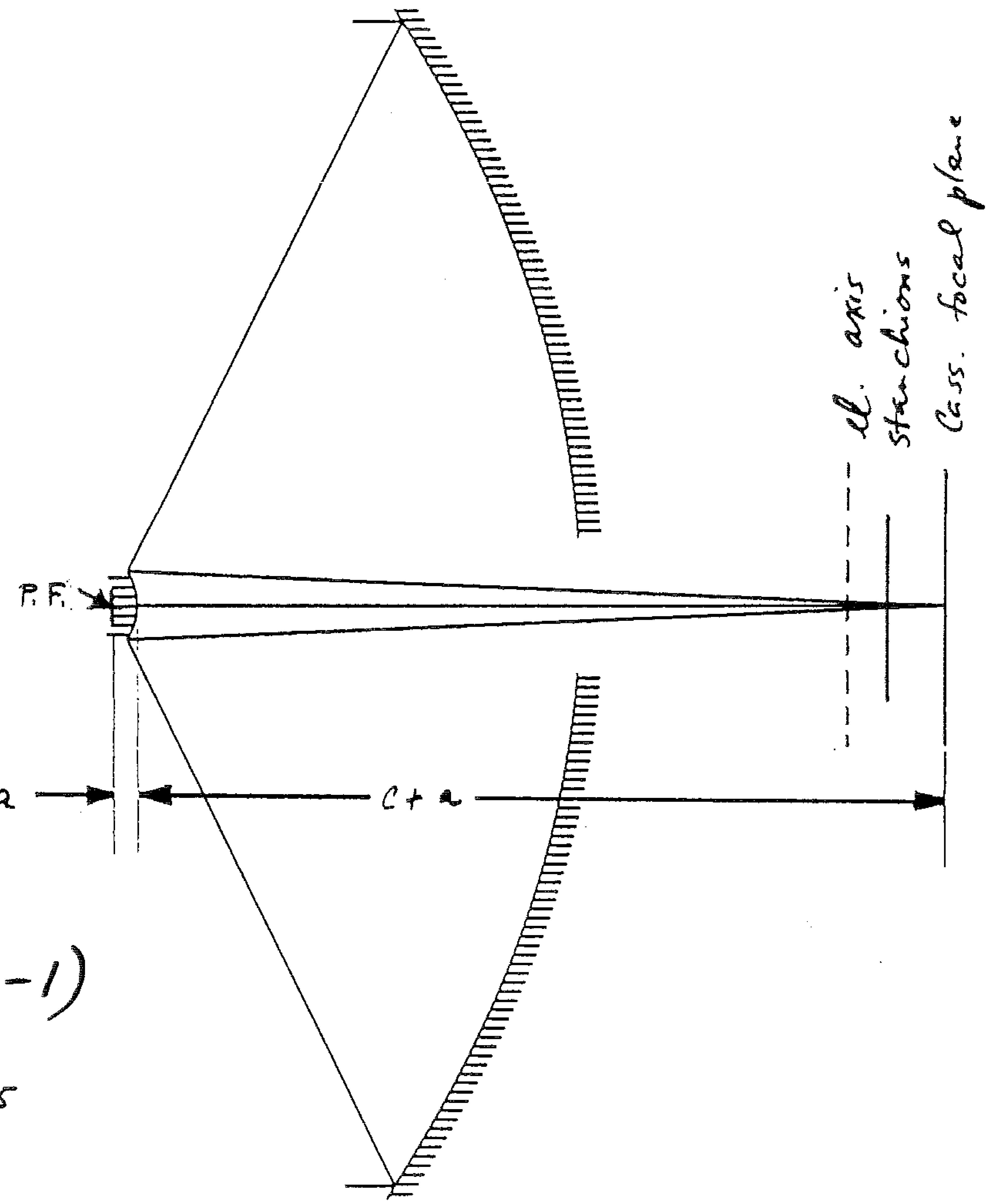
1. D.J. Schroeder 1987, *Astronomical Optics*.
2. M. Born and E. Wolf 1980, *Principles of Optics*.
3. E. F. Erickson and S. Matthews 1975, NASA TM X-3137.



1
0.9
0.8
0.7
0.6
Strehl ratio
0
0.02
0.04
0.06
0.08
0.1
rms wavefront error/λ

PRIMARY:		
eccentricity	=	1.0
K	=	-1.0
Focal length	=	162.333 in
	=	4123.2582mm
Diameter	=	10400.0 mm
Pri. Illumination	=	10227.8 mm
Primary to chop. sec. vertex separation	=	3889.4367 mm
CHOPPING SECONDARY:		
diameter	=	580.0 mm
radius of curvature	=	483.6119 mm
c-a	=	233.8215 mm
c+a	=	7081.1785 mm
eccentricity	=	1.0682954
K	=	-1.1412551
Cass. focal ratio	=	12.36
CASSEGRAIN AREA		
elev. axis to focus	=	863.6 mm
	=	34.00 in
stanchions to focus	=	mm
	=	in

558.8 mm = 22 in



secondary eqn: $z = a\left(\sqrt{1+\frac{r^2}{c^2-a^2}} - 1\right)$

magnification: $m = \frac{c+a}{c-a} = 30.285$

$f_{eq} = m f_{pri} = 124.87$ m

2000 MM

CSO chopping secondary optics layout 26-DEC-91

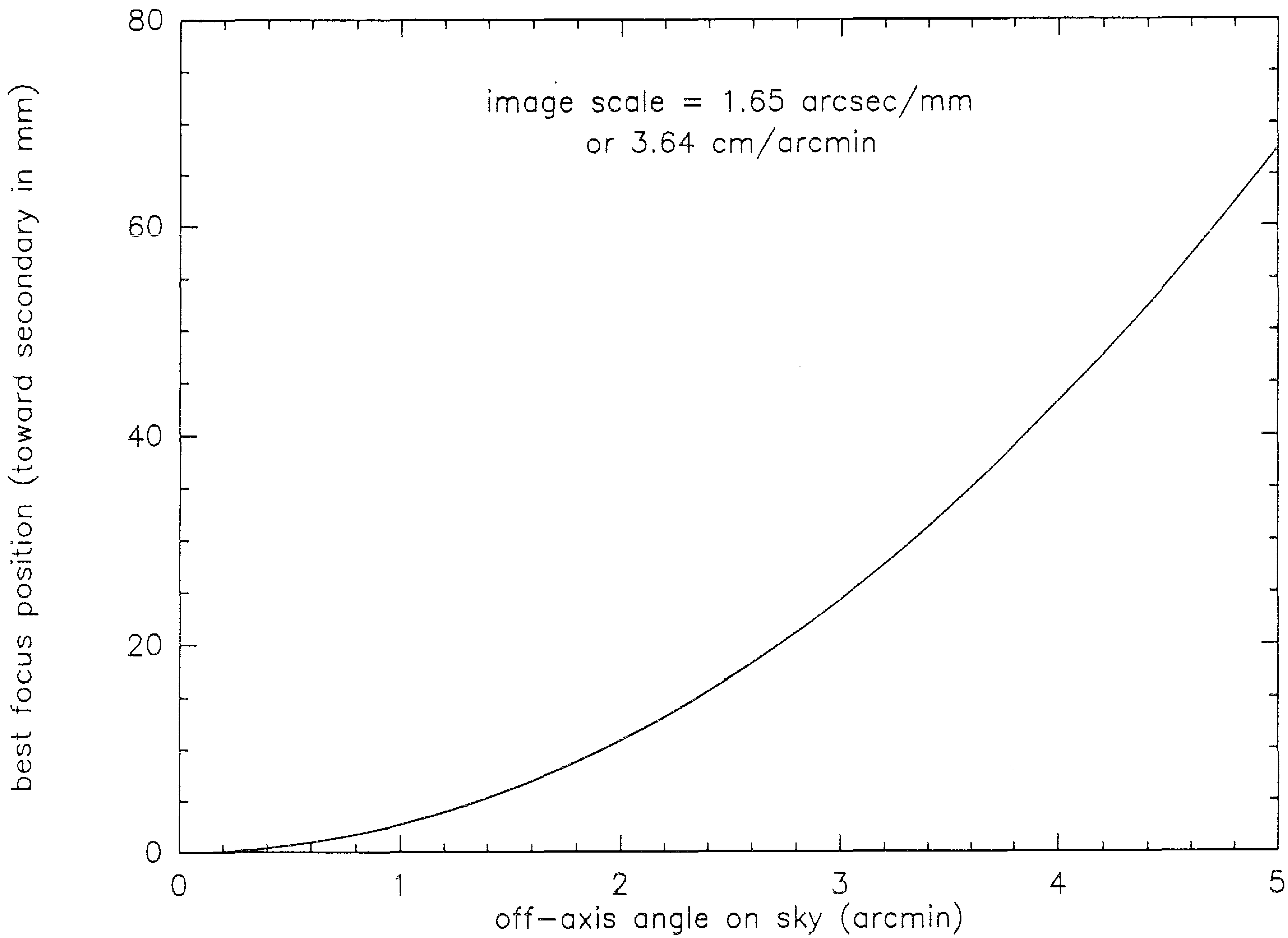
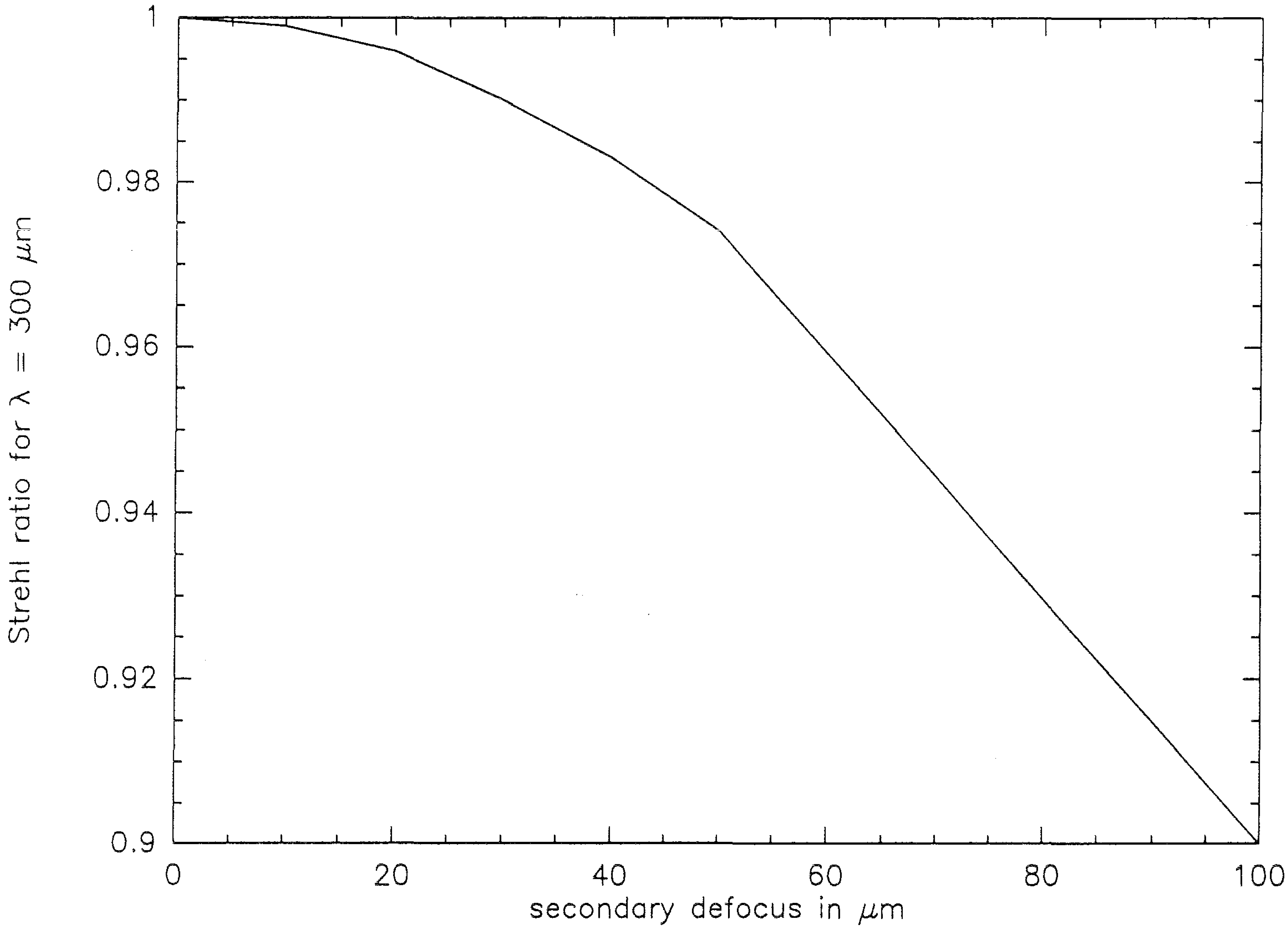
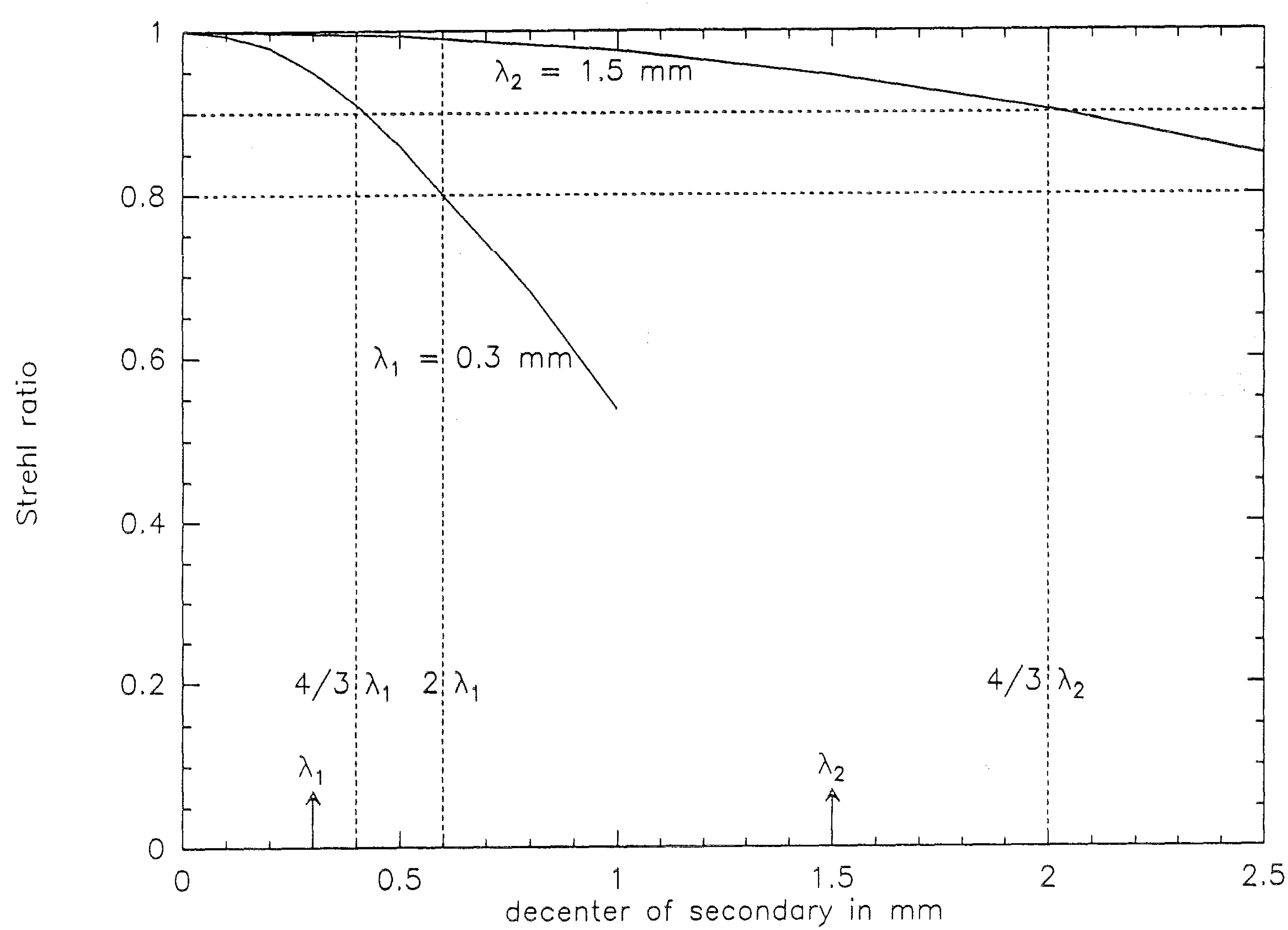


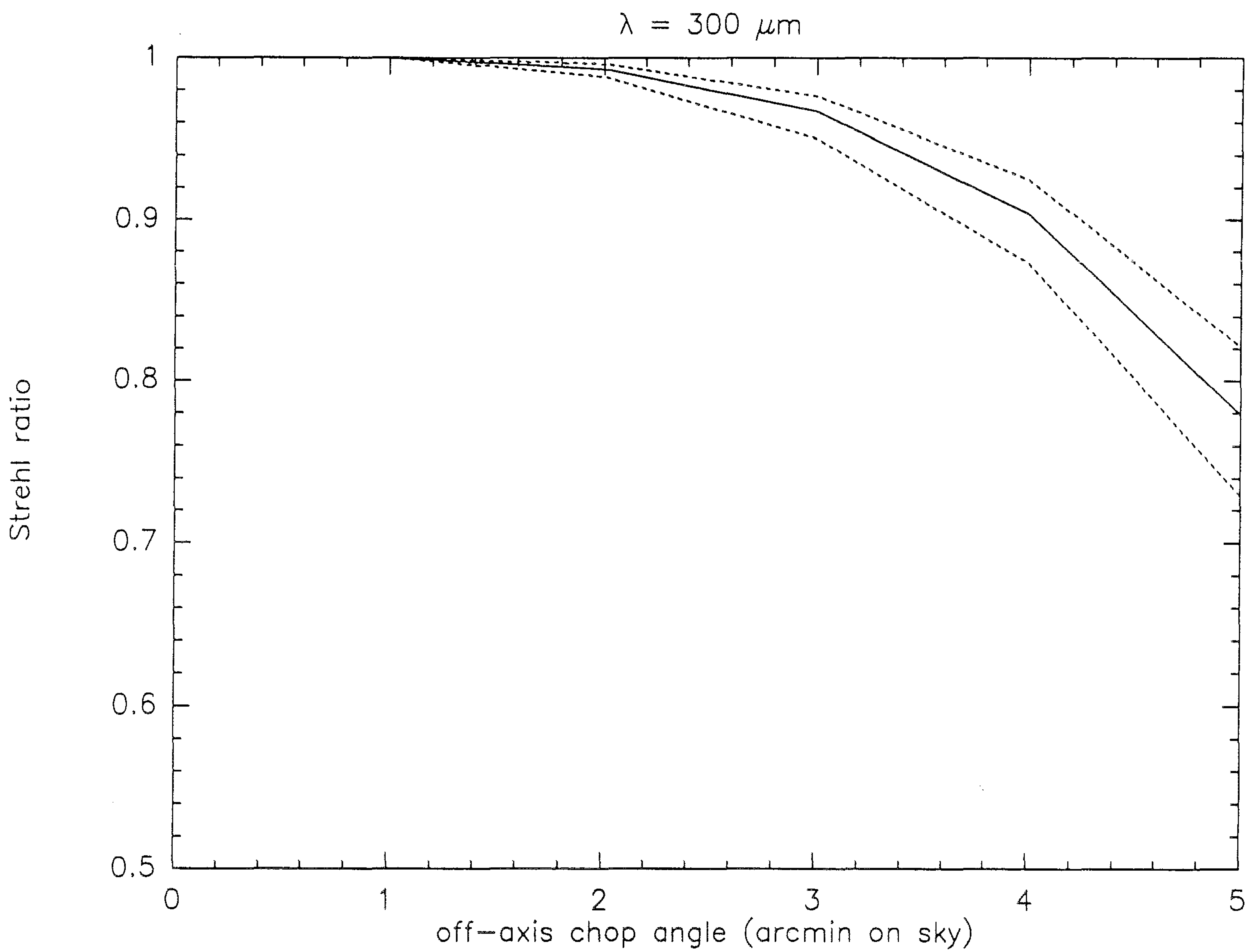
image scale = 1.65 arcsec/mm
or 3.64 cm/arcmin
best focus position (toward secondary in mm)
off–axis angle on sky (arcmin)
0
20
40
60
80
0
1
2
3
4
5



1
0.98
0.96
0.94
0.92
0.9
0
20
40
60
80
100
secondary defocus in μm
Strehl ratio for λ = 300 μm

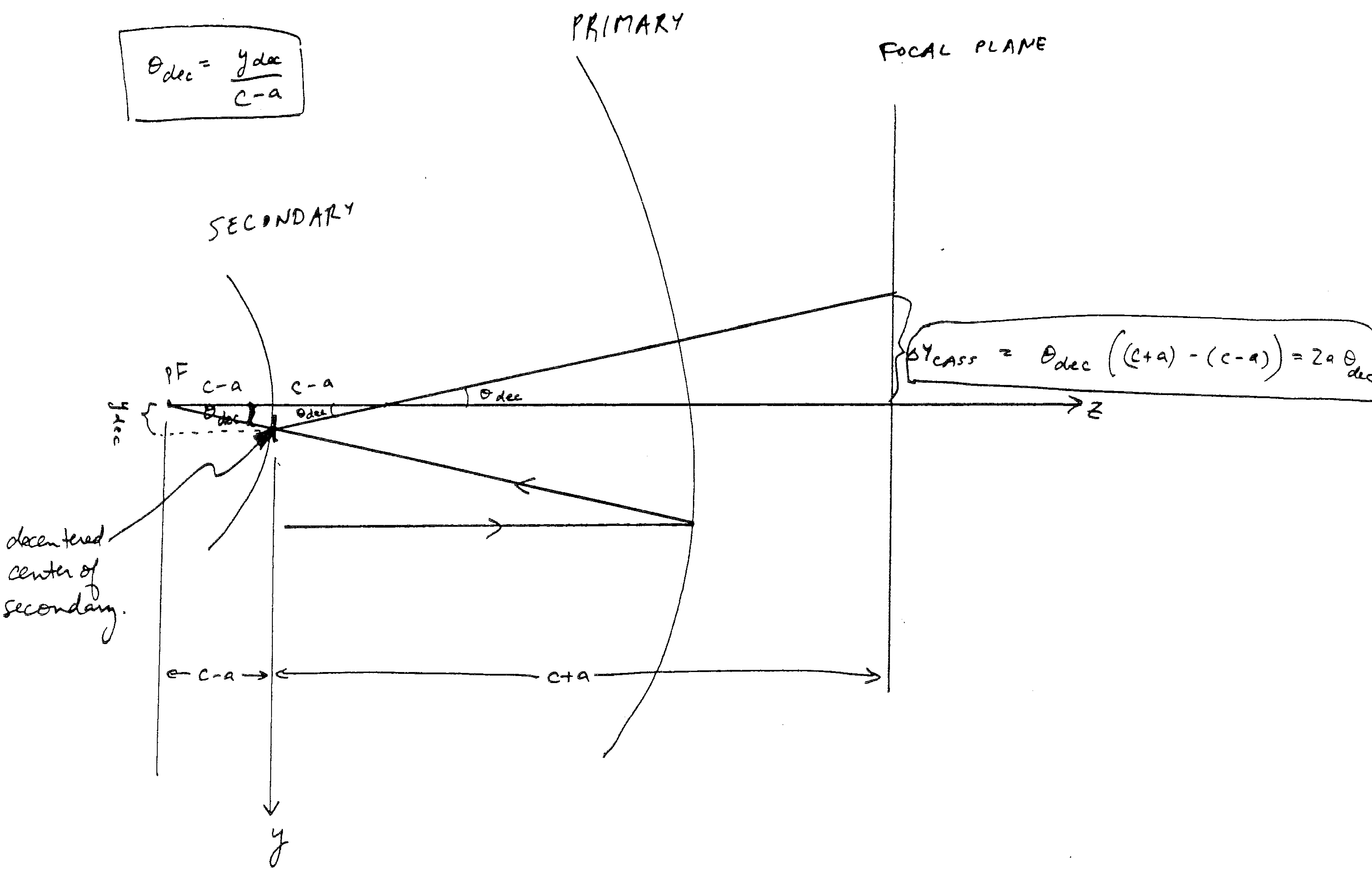


Strehl ratio
decenter of secondary in mm
λ2 = 1.5 mm
λ1 = 0.3 mm
4/3 λ1
2 λ1
4/3 λ2
λ1
λ2
0
0.2
0.4
0.6
0.8
1
0.5
1.5
2
2.5



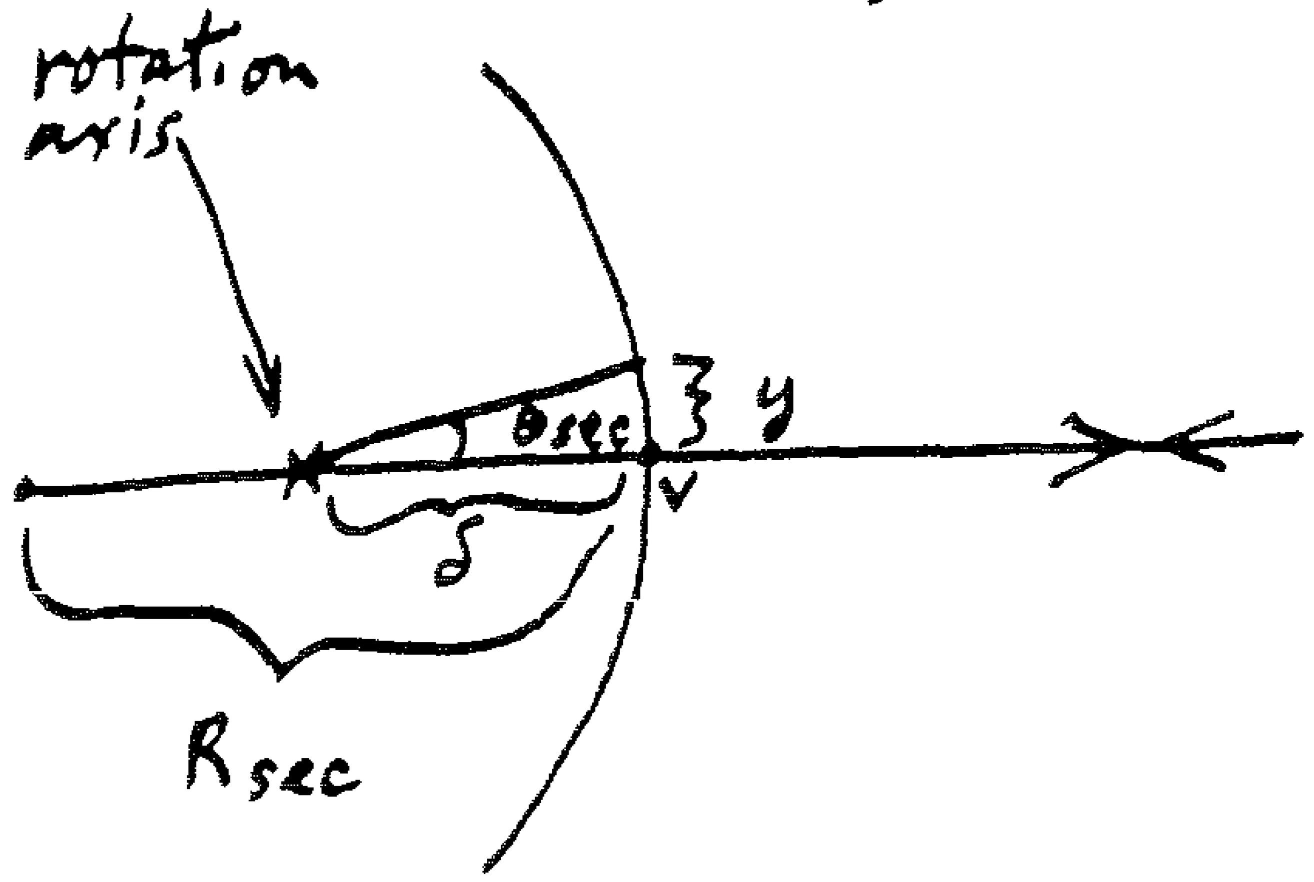
λ = 300 μm
Strehl ratio
1
0.9
0.8
0.7
0.6
0.5
0
1
2
3
4
5
off–axis chop angle (arcmin on sky)

SECONDARY DECENTER



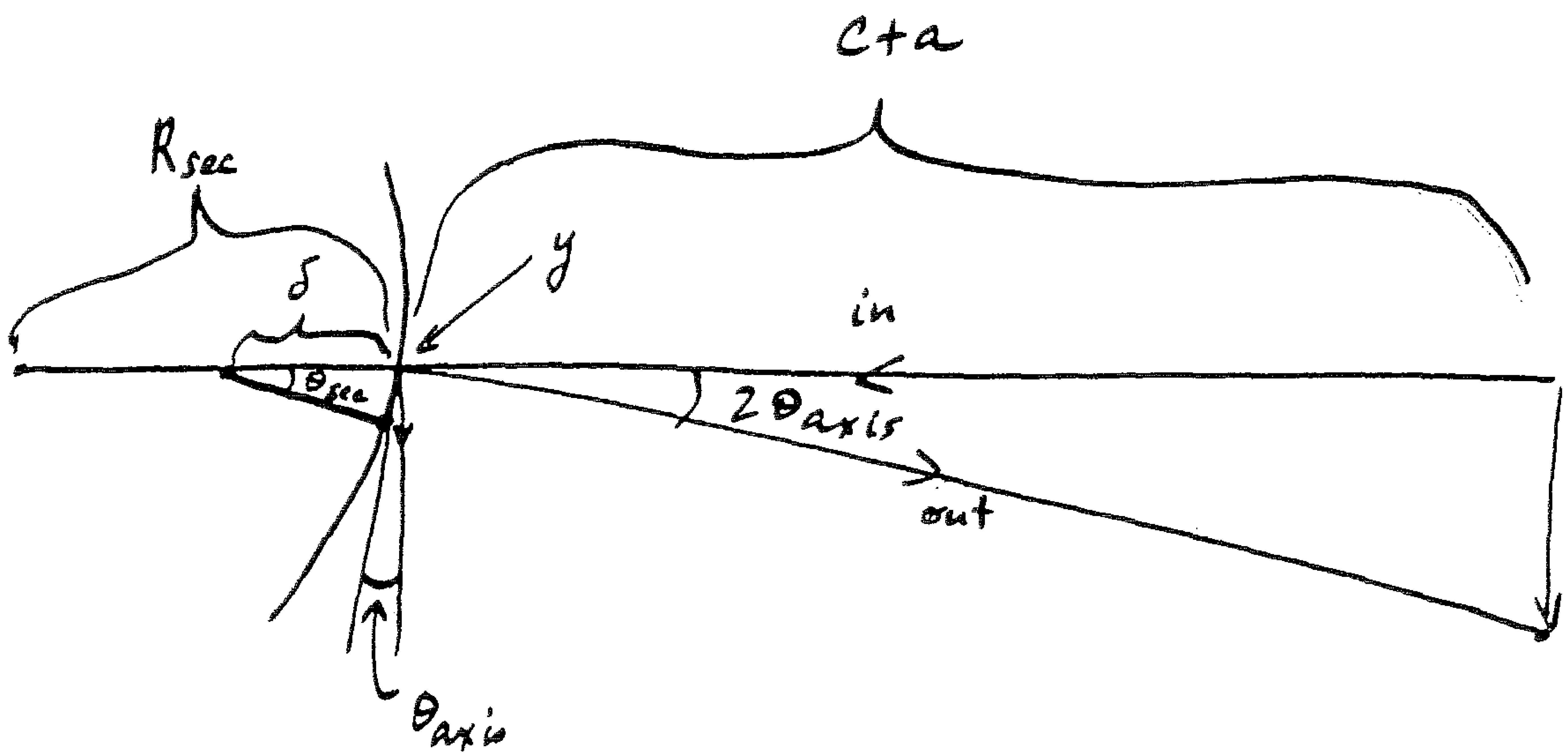
Appendix : Secondary Rotation

Before Rotating Secondary



(v = vertex)

After Rotating Sec. so that y is now on axis.



$$\Delta y_{cass} = 2\theta_{axis}(C+a)$$