NIGHT and DAY

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George W. Bunton

Some people say they can tell time by the Sun,
but I can't see the numbers!
.... Anonymoous 5th grader

The diurnal habits of mankind, like those of most other creatures of the Earth, have been derived in an environment influenced by the rotation of the Earth under the illumination of the Sun. Even in this modern day, when artificialities have greatly modified the natural environment, humans still regulate their lives largely by the rising and setting of the Sun. This is more true in rural than in urban communities where artificial daylight has been extended far into the night.

Almost all the flora and fauna of the Earth are light dependent, and many of the foraging animal species have developed visual ability under very feeble illumination . While the Sun is the the primary luminary of the Earth, there are other natural sources that provide enough light to accommodate some creatures. The full Moon is about one twenty-four thousandths as bright as the Sun, but the human eye can accommodate to that reduced light to permit one to read under a full Moon. When the Moon is not present, some light is provided by the stars, and even the upper atmosphere itself glows faintly at night. This airglow phenomenon has been observed and photographed by astronauts as a thin luminescence gracing the dark curve of the Earth where there was no solar illumination.

Many of the wonders of nature are beyond the appreciation of most of us, immersed as we are in the cocoon of civilization.

This dissertation attempts to describe and explain the various kinds of time used in astronomical as well as civil affairs, and to elucidate the relations among them. Practical applications include definitions and computations of twilight, sunrise, and sunset, among others, as well as their seasonal and geographical variations.

TIME

The period called the *day* is the time for the Earth to rotate once upon its axis with respect to the Sun. It has arbitrarily been divided into twenty-four hours. The twenty-four hour division has led to a very practical geographical division of the Earth into twenty-four zones, east and west, called *time zones*. The rotating Earth causes the Sun to appear to move westward, covering one time zone each hour. North-south lines, called *standard meridians*, are described as passing centrally through each time zone. The standard meridians do not form the boundaries of the zones, but the boundaries lie half-way between these meridians. For political convenience, the boundaries of the time zones are often distorted east or west to include neighboring communities within the same time zone.

During the period of British maritime dominance, the *zero*, or *prime meridian* was designated as passing through the observatory at Greenwich, England. The standard meridians then are numbered east and west from there to the one on the opposite side of the Earth, called the *international date line*.

The system is fundamentally sound, for it permits travel throughout the world under standardized time conditions. All clocks within a time zone read the same under these standardized time conditions and that time is called the *standard time* of that zone. When one moves from one time zone to an adjacent one, the time must be changed by a full hour. If there were no time zones, as was once the case, each community would set its own time based on the passage of the Sun at that particular location. This, of course, led to a great deal of confusion until time zones were agreed upon. The need for such standardization occurred with the development of the railroads which quickly spanned the continents.

SUNDIAL

A sundial is a simple instrument for reading time during daylight hours from the position of the Sun. However , comparison of *sundial time*, also called *apparent time*, with standard time usually shows a considerable difference, and the difference increases and decreases during the year. The rate of rotation of the Earth is remarkably uniform, varying only a tiny fraction of a second per year, but there are other factors in the Earth's motion that influence sundial time.

When placed on a standard meridian, the sundial will agree with a standard time clock on only four days of the year, running alternately ahead of and behind the standard clock. Since the standard clock is assumed to run at a uniform rate, the conclusion must be that the Sun does not move across the sky at the same rate throughout the year.

The standard clock keeps the time of an imaginary Sun, called the *mean Sun*, which is assumed to move uniformly throughout the year. Standard time is sometimes called *mean time*, referring to the mean Sun. The difference between the standard clock and the sundial is the same from year to year on the same date. The difference is tabulated as the *equation of time*. The equation of time is usually defined (and will be so used in this article) as:

EQUATION OF TIME = LOCAL APPARENT SOLAR TIME (sundial time) minus LOCAL MEAN SOLAR TIME (standard time).

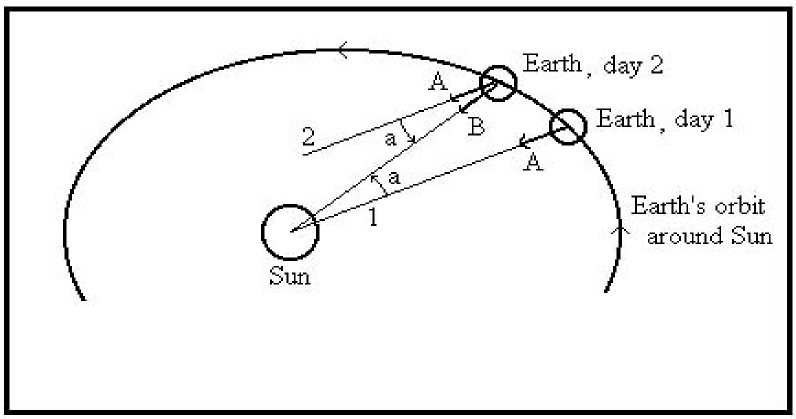
Thus, if the sundial is ahead of the clock (reading later), the equation of time is positive, and if behind (reading earlier), it is negative. The equation of time is often displayed graphically on terrestrial globes as the narrow figure-8 curve, called the *analemma*.

ELLIPTICAL ORBIT

The reasons for this yearly variation in the apparent motion of the Sun are twofold. The first reason has to do with the fact that the Earth's orbit is not a perfect circle, but is elliptical with the Sun being nearer one end of the ellipse. The speed of the Earth in this elliptical orbit varies from a minimum at the farthest distance to a maximum at the closest distance of the Earth to the Sun. The second reason for the yearly variation has to do with the fact that the Earth's equator is inclined to the plane of the Earth's orbit around the Sun. These two effects are explained in the following paragraphs.

1. **Elliptical Orbit**. While the Earth is rotating upon its axis, it is also moving around the Sun in the same sense, or direction, as its rotation. If we select a spot on the Earth where the Sun is directly overhead, in order for that spot to rotate with the Earth and come back so that the Sun is overhead again, it must turn a little extra because of the Earth's motion around the Sun. The Earth turns a little more than once with respect to the stars in order to complete one rotation with respect to the Sun. The "little extra" is just the angle through which the Earth has moved around the Sun in a day's time. On the average, this angle amounts to a little less than one degree per day (360 degrees/ 365 ¼ days) and is illustrated in Figure 1.

Figure 1. *The Earth must rotate 360 degrees plus* a, *a very small angle, for observer at A to return to the same position relative to the Sun at B.*

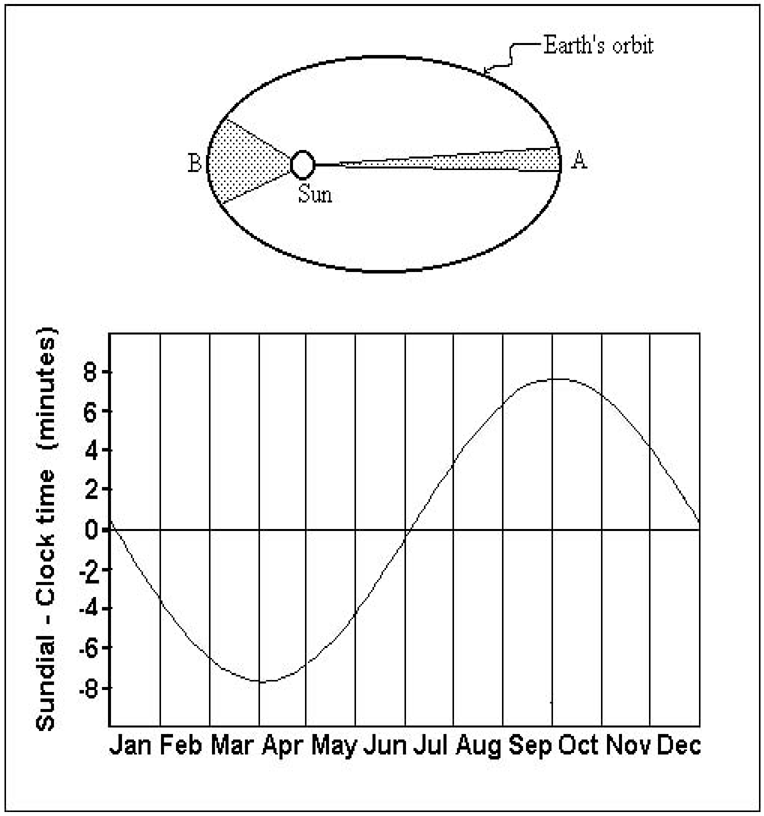


The time for the Earth to turn this small angle is about four minutes. This little difference would cause no concern if it were always the same, but it is not! Recalling that the Earth moves in an elliptical path (much exaggerated in Fig. 1) around the Sun, rather than a circular path, it turns out that the Earth is nearer to the Sun in January than in July. The difference is about three million miles (out of an average distance of ninety-three million miles). The speed of the Earth in its orbit increases as it gets nearer to the Sun. Since the Earth is closest to the Sun in January and furthest in July, it follows that the Earth is moving more rapidly in its orbit in January than in July! Thus, the Earth must rotate a little more each day from October to April to return to a chosen spot to face the Sun again. This small amount each day accumulates until it amounts to a difference of 7.7 minutes on April 2. Having to turn a little more each day means the sundial lags behind the standard clock and so the sundial time minus standard time on April 2 is -7.7 minutes. From April 2 on, the Earth rotates a little less each day to return to a chosen spot to face the Sun again, and this decrease accumulates from April to October until it amounts to a difference of +7.7 minutes on October 2. The difference between sundial time and clock time resulting from the varying speed of the Earth in its orbit is graphically illustrated in

Figure 2

Figure 2. (top) The Earth moves slowest at A and fastest at B.

(bottom) Equation of Time component due to the eccentricity of the Earth's orbit.



INCLINATION OF THE ECLIPTIC

The second reason for the yearly variation of the Equation of Time has to do with the fact that the Earth's equator is inclined to the plane of the Earth's orbit around the Sun.

2. **Inclination of the Ecliptic**. Another element enters the scene, causing the sundial to vary from the clock. This effect is purely a geometrical one. The axis of rotation of the Earth is not perpendicular to the plane of its orbit around the Sun, but is tilted by an angle of 23½ ^{o} . So, as the Earth revolves around the Sun, the north pole is tilted 23½ ^{o} toward the Sun on June 21, and 23½ ^{o} away from the Sun on December 21, as illustrated in Fig. 3. These are the dates of the *summer* and *winter solstices* as recognized in the northern hemisphere. The result, as seen from the northern hemisphere, is that the Sun crosses the sky at noon much higher in June than in December, and if one were to plot the path of the Sun during the year, as seen against the background of the stars, it would appear as a line crossing over the celestial equator on March 21 and September 21 the *vernal* and *autumnal equinoxes*. The annual apparent path of the Sun against the background of the stars, called the *ecliptic*, is shown in Fig. 4, along with the celestial equator. The celestial equator is an imaginary line in the sky directly above the Earth's equator. Thus, when the Sun is on the celestial equator, it stands directly above the Earth's equator We see that the

path extends north and south > of the equator by 23½ ^{o}.

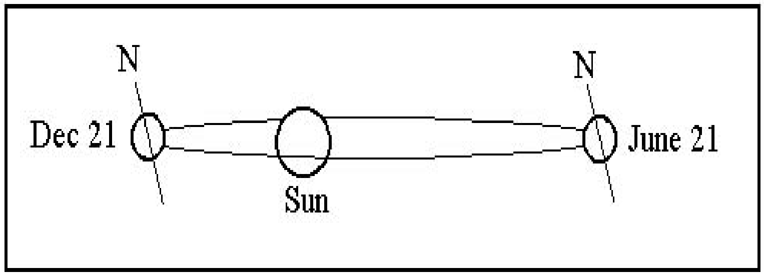


Figure 3. Inclination of the Earth's axis with respect to the plane of its orbit.

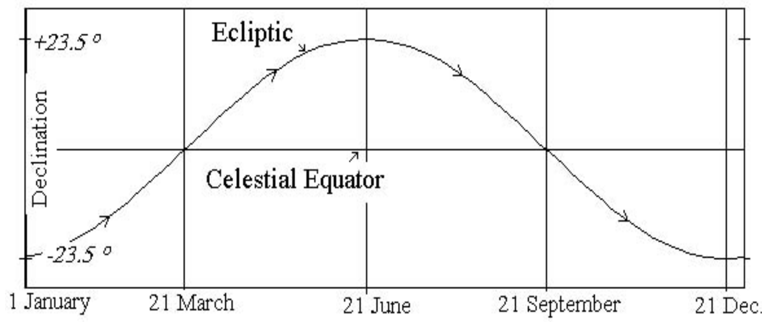


Figure 4. The annual apparent path of the Sun, the ecliptic, reaches 23½ ^{o} north and south of the celestial equator.

Ignoring the change
of speed of the Earth in its elliptical orbit (effect number one above), the true eastward motion of the Sun is greatest when all of its motion is due eastward. This occurs in June and December. In March and September, part of the Sun's motion is northward or southward, and the eastward part of its motion is reduced. This makes the sundial fall behind the standard clock at the solstices and move ahead of the clock at the equinoxes. Fig. 5 illustrates this geometrical effect upon the equation of time.

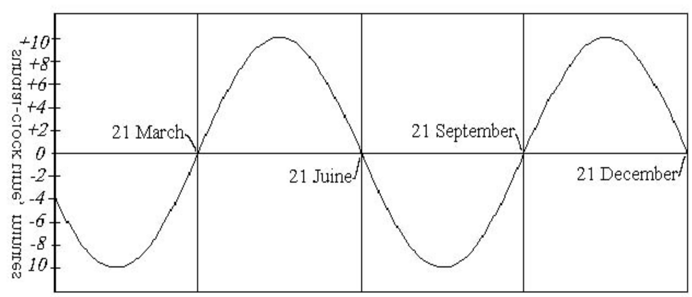


Figure 5. Equation of Time component due to the obliquity of the ecliptic. (obliquity = 23½ ^{o}).

EQUATION OF TIME

In the two previous sections, we have seen how the difference between sundial time and standard time depends on two effects: the eccentricity of the Earth's orbit and the inclination of the Earth's orbit.

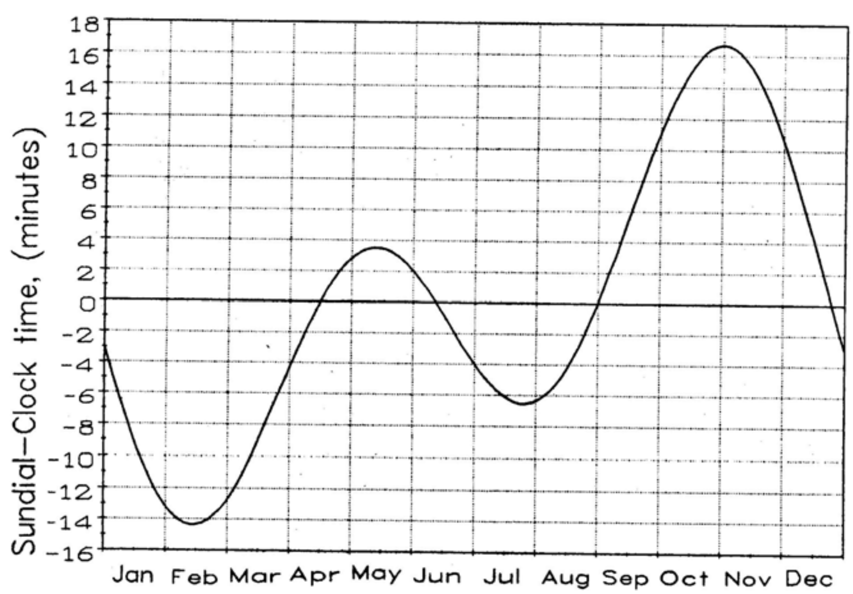
The combination of these two effects, which is the true equation of time, is plotted in Fig.6. In December and January these two effects are both working to slow the sundial time, while in June and July the two effects are opposed to each other. The sundial lags only six minutes during June when the two effects are opposed, but lags 13½ minutes during December. The equation of time expresses the relationship between the sundial and standard time, and the standard time is then available from the sundial by applying the proper value, plus or minus, from the equation of time. But such conversion yields true standard time only if the sundial is on the standard meridian. One must know one's distance east or west of the standard meridian in order to make the remaining correction to the sundial time.

The Earth turns through one time zone in an hour. The time zone is 15 degrees wide (one twenty-fourth of 360 degrees), so each degree of longitude within the time zone is equivalent to four minutes of time (60 min./15^{o}). This then is the correction to make for each degree of longitude away from the standard meridian: minus if east or plus if west of the standard meridian

As an example, suppose that you are located at longitude 155 degrees west. What is the correction to arrive at standard time for your time zone? The standard meridian is the 150 degree west meridian, so you are located 5 degrees west of that. Every degree is 4 minutes of time, so the sun passes overhead at your longitude 4 x 5 = 20 minutes later than at the standard meridian. Thus, you must add 20 minutes from your sundial time to get the standard zone time. This, of course, is in addition to the time that must be added or subtracted according to the equation of time. See Appendix B for additional examples.

Figure 6

The Equation of Time



SUNRISE and SUNSET

When the clock is gaining on the sundial, the Sun rises and sets later each day, and when the sundial is gaining on the clock, the Sun rises and sets earlier each day. If the two effects which give us the equation of time were solely responsible for sunrise and sunset times, these times would be late in summer and winter and early in spring and fall. Most of us would say at once that, of course this is not true. But it *is* true for anyone living on the equator!

On a standard meridian at the equator one might expect the Sun to rise at 6:00 A.M. and set at 6:00 P.M., but the Sun rises at 6:03 A.M. in July, a summer month, and also rises late, at 6:11 A.M. in February, a winter month. It rises seven minutes before 6:00 A.M. in mid-May, and 20 minutes before 6:00 A.M. at the end of October. At the equator these effects are entirely accounted for by the equation of time.

The daily path of the Sun as seen at the equator on the first day of spring, summer, fall, and winter is illustrated in Figures 7 and 7a . At the equator the Sun rises perpendicularly from the horizon and sets perpendicularly, regardless of the season. Also, the total path of the Sun, day and night, is divided equally by the horizon. There are always twelve hours of daytime and twelve hours of night-time at the equator, except for two minor effects that increase daytime by about eight minutes. First, since we mark the instant of sunrise as the time the Sun's upper edge or "limb" just touches the horizon, the actual center of the Sun is still below the horizon by half the diameter of the Sun, 16 arc minutes or ¼ degree.. It will take an additional minute for the Sun's center to be on the horizon. At sunset the same thing happens and so an additional two minutes are gained for daytime. Second, when the Sun's limb appears at the horizon, it is actually still 43 arc minutes below the horizon but only appears to be at the horizon due to the refraction or bending of the Sun's rays by the Earth's atmosphere. This effect causes the sunrise to appear about three minutes early and sunset late by the same amount. Taking both effects together, the length of daytime is about 8 minutes more than 12 hours, and so, of course, night-time will be 8 minutes less than 12 hours, resulting in daytime being 16 minutes more than night-time at the equator, or for that matter, anywhere during the equinoxes (March 21 and September 21).

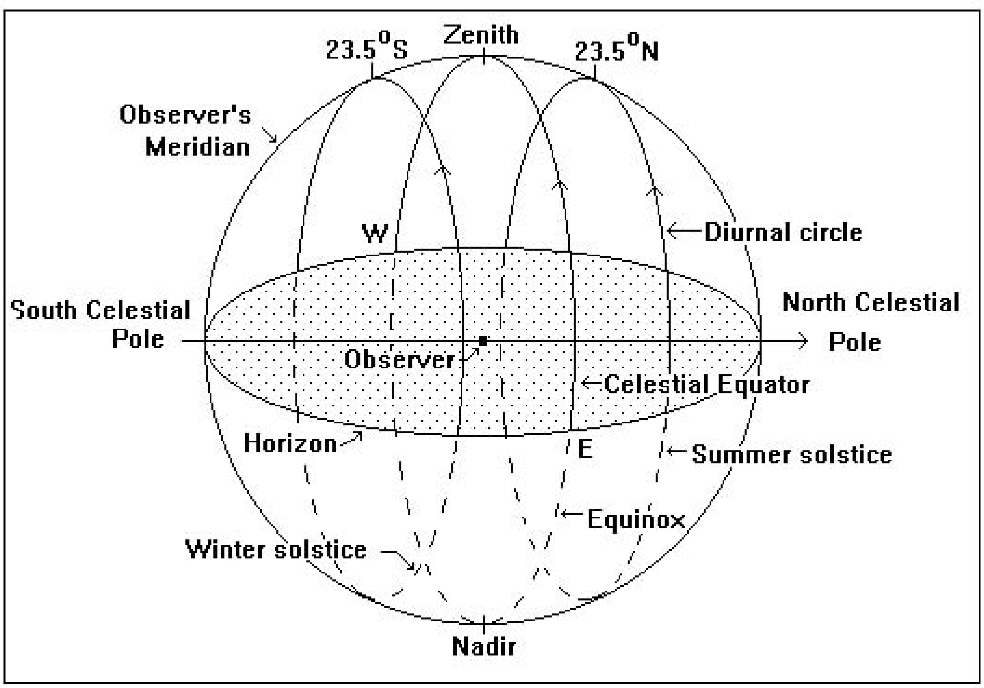


Figure 7. The daily or diurnal paths of the Sun during the solstices(21

December and 21 June) and the equinoxes (21 March and 21 September) as seen by an observer at the equator. Solid lines are daytime, dashed lines are night-time. At all seasons on the equator, the daily paths of the Sun are divided equally above and below the horizon.

The same information shown in Figure 7 is presented in Figure 7a, below, in the form of a polar plot of the position of the Sun in the coordinates of the azimuth and altitude of the Sun as seen by an observer at that latitude.

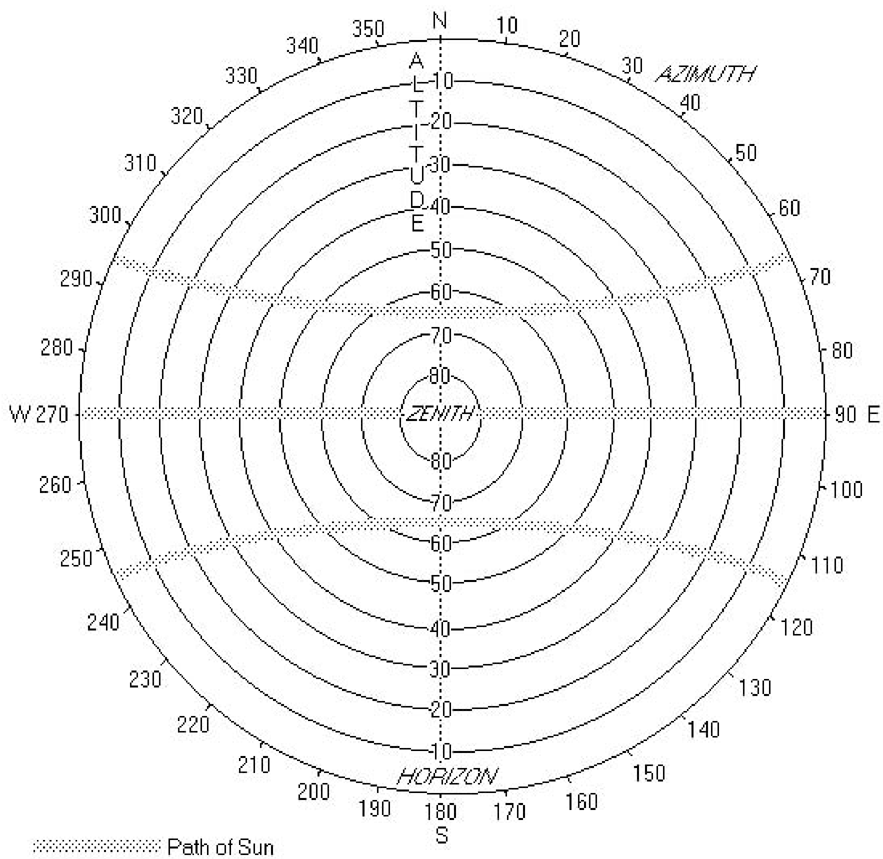


Figure 8 and 8a show the apparent paths of the Sun as seen from Hawai'i, the southernmost State of the United States, 21 degrees north of the equator. The paths are all parallel to each other, but are slanting at 21 degrees to the horizon. It will also be noticed that the horizon divides the total path of the Sun into equal periods only on the first days of spring and fall, i.e., the equinoxes. In summer, the portion of the Sun's path above the horizon is much greater than the night portion, and the reverse is true in the winter. This illustrates the *geographical effect*, which depends on the observer's latitude.

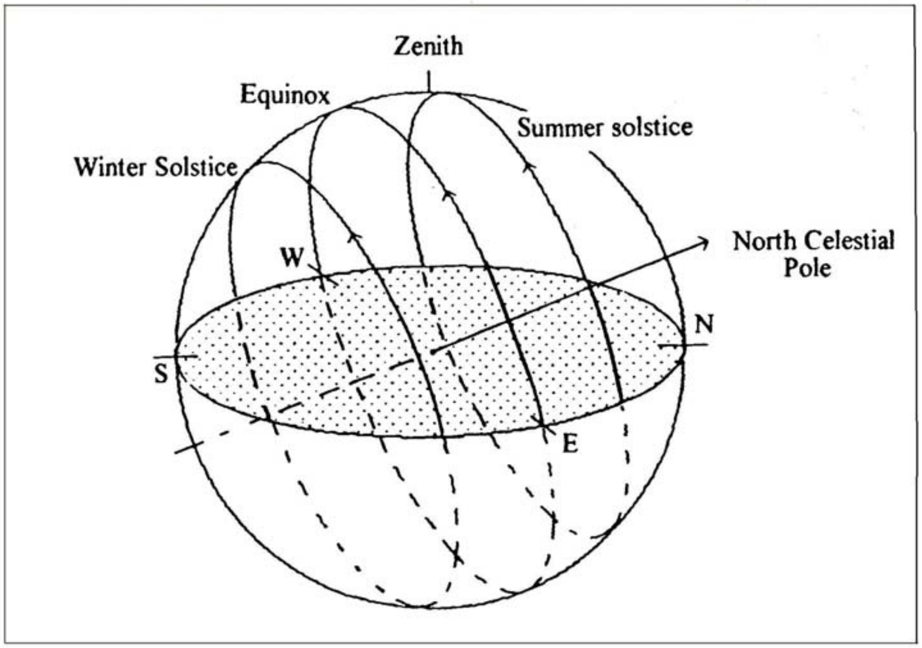
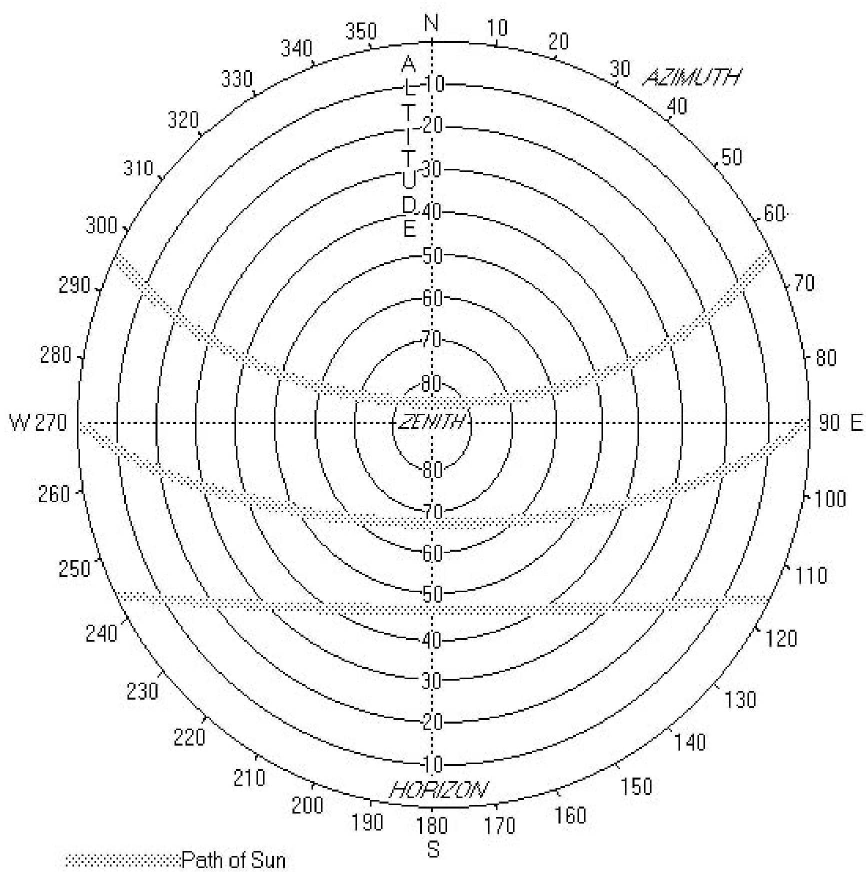
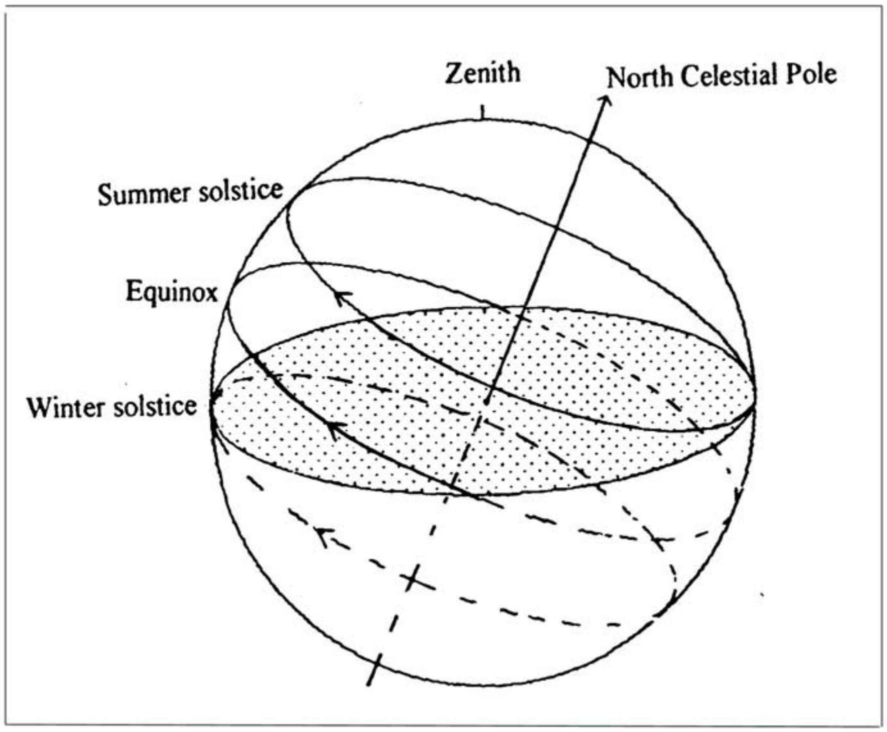


Figure 8 and 8a (below). The daily path of the Sun as seen from Hawai'i on the first day of spring, summer, fall, and winter.

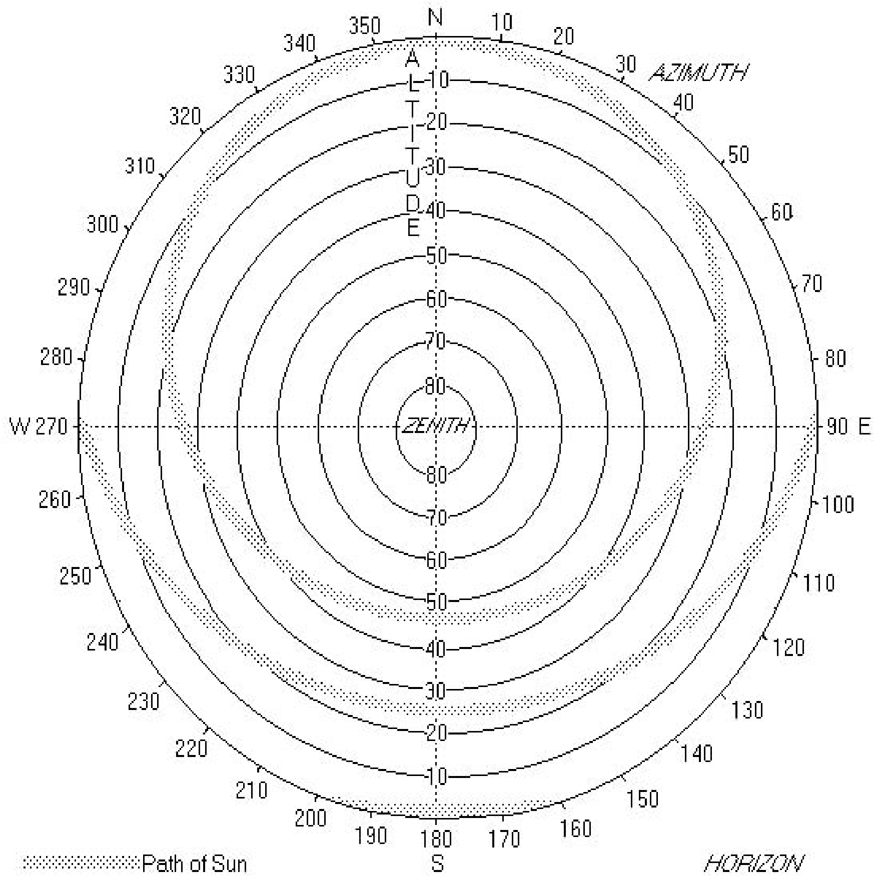


An extreme situation is shown in Figures 9 aand 9a for a location at the Arctic Circle, latitude 66½ ^{o} north. The Sun is above the horizon all day at the beginning of summer, barely touching the horizon at midnight.

At the beginning of winter the Sun's path is entirely below the horizon. This latter situation is modified by the refraction of sunlight by the Earth's atmosphere which causes the Sun to appear a little higher at the horizon than it actually is. Because of this refraction, the Sun appears briefly above the southern horizon at noon on the first day of winter at the Arctic Circle.



Figures 9 and 9a (below). The daily path of the Sun as seen at 66.5 degrees north latitude (the Arctic Circle) on the first day of spring, summer, fall, and winter.



TWILIGHT

For a time before sunrise and after sunset, light from the Sun illuminates the atmosphere to produce some skylight, known as *twilight*. Twilight is arbitrarily divided into three increments. The period when the Sun is 6 degrees or less below the horizon is called *civil twilight* . The time of the end of civil twilight is often published in newspapers along with the time of sunrise and sunset. When the Sun is between six and twelve degrees below the horizon, the period is called *nautical twilight*. During this period it is dark enough to see the brighter stars, used for navigation, and still light enough to see the horizon. When the Sun is between twelve and eighteen degrees below the horizon, the period is called *astronomical twilight.* A truly dark sky suitable for sensitive astronomical observations is possible only after astronomical twilight.

Further inspection of figures 7, 8, and 9 reveals that the time for the Sun to reach these twilight angles differs at different seasons. At the equator, the evening Sun reaches the position ending civil twilight in only 23 minutes. In Hawai'i, civil twilight lasts 27 minutes in winter and 28 minutes in summer. At 45 degrees latitude the same period is 35 minutes in winter and 37 minutes in summer.

The more popular concept of twilight more nearly matches the period of nautical twilight, and the effect of latitude is more pronounced for this period. In Hawai'i, nautical twilight ends 52 minutes after sunset in winter and 55 minutes after sunset in summer. At 45 degrees latitude, the corresponding periods are 73 minutes in winter and 85 minutes in summer. the longer hours of twilight at higher latitudes contribute to the acceptability of daylight saving time. In London, England, 51 degrees north, on the first day of summer, the Sun sets at 8:18 P.M., and nautical twilight ends nearly two hours later, at 10:16 P.M. Only a little farther north, latitude 54½ o , nautical twilight runs through midnight, lasting all night long in summer. At the Arctic Circle, while the Sun is below the horizon for long periods in the winter months, much of its path is within the nautical twilight zone below the horizon , so that twilight lasts most of the day even on the first day of winter. The portion of night-time hours that receives some illumination, through nautical twilight, is shown in Fig. 10.

Figure 10. Seasonal and geographic variations in daylight, twilight, and dark night hours.

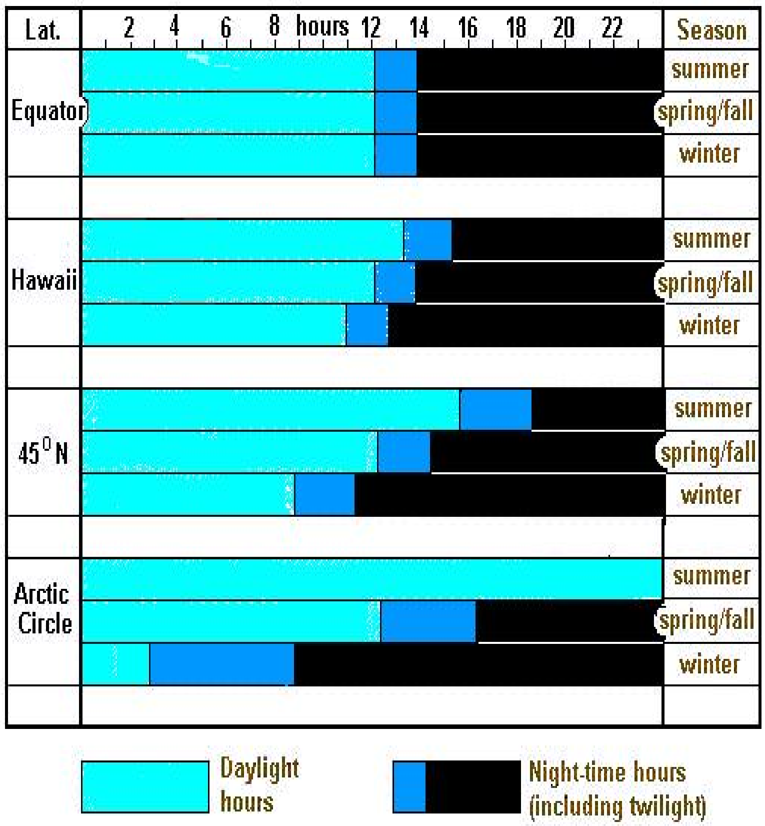


Figure 10 also summarizes the results of the previous section on Sunrise and Sunset. The larger seasonal change in daylight hours at 45 degrees latitude over that at the latitude of Hawai'i, reveals the advantage of "daylight saving time" in northern cities where the summer daylight period is so long. In Hawai'i, the daylight period is more nearly the same all year around. Shifting the clocks in Hawai'i can only rob the short morning hours to give extra daylight in the evening. Hawai'i does not have the morning hours to spare. At the time of this writing, Hawai'i does not observe "daylight time."

A seemingly contradictory condition in the time of sunrise is often noted by critical observers. In the northern hemisphere the shortest day of the year is the first day of winter, December 21. Yet the time of sunrise continues to grow later into early January when the duration of the daylight period is actually lengthening. The reason is that this is the time of the year when the equation of time still dominates the seasonal effect, causing both sunrise and sunset to occur later each day. This effect is greater for locations near the equator. In Hawai'i, at 21½ o north, the latest sunrise is January 15, twenty-five days after the winter solstice, the shortest day of the year. At 45 degrees north latitude, the latest sunrise occurs on January 3, and in London, at 51 degrees north, the latest sunrise is on December 31. The annual variations in sunrise and sunset times at these latitudes are illustrated in Fig. 11. The large differences in seasonal times of sunrise and sunset as well as differences at different latitudes may be seen in this figure . It may be noted that, at higher latitudes, the observation of "daylight saving time" is of far greater value than nearer the equator. Furthermore, the additional morning and evening twilight greatly extends the total "daylight" at higher latitudes.

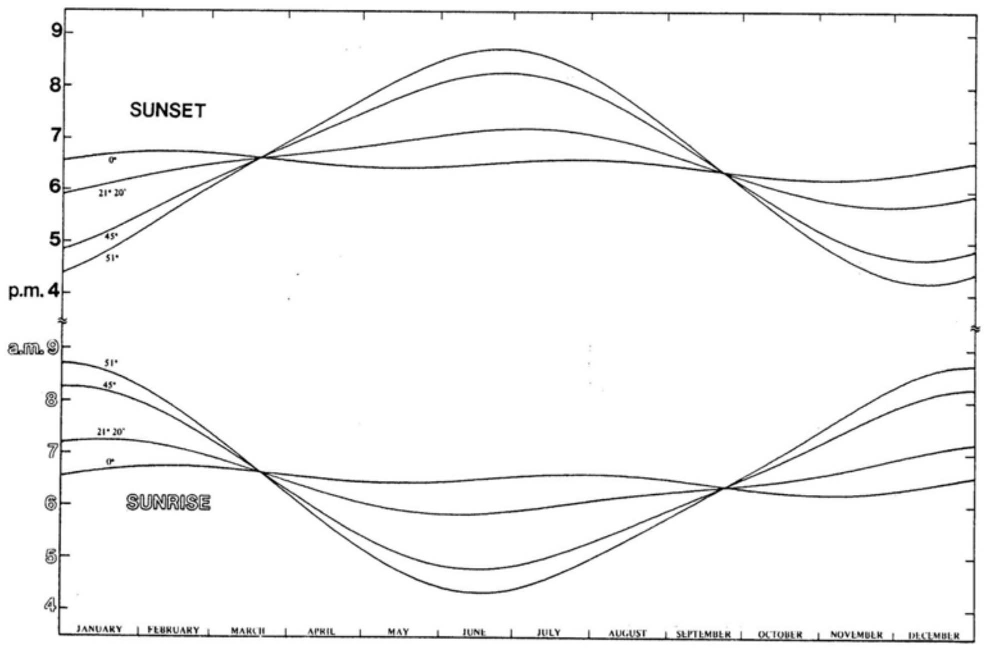


Figure 11. The annual variations in sunrise and sunset times at various locations: 0^o (equator), $21^o 20'$ (Honolulu), 45^o (Minneapolis), 51^o (London).

EXERCISES

The Equation of Time (ET) allows one to calculate the Standard Time at which the Sun will be found in a particular part of the sky. For example, if it is asked at what time does the Sun rise in Hilo, Hawaii, on March 21, we must know the ET on March 21 and the longitude of Hilo. The ET can be read from the graph and is approximately -8 min. . The longitude of Hilo is 155o W, corresponding to the example under Equation of Time. First, let us determine the time of sunrise at the Standard Meridian of 150^o W, and then make the correction for Hilo's longitude. The apparent time at sunrise must be 4 minutes before 6 a.m. (The explanation of the 4 minutes is found under Sunrise and Sunset). From the definition of ET = Apparent time - Mean (or Standard) time, we see that the Standard time in this case is the App. time - ET, or, 5h 56min - (-8 min) = 6h 4min. This is now corrected for Hilo's longitude by adding 20 minutes, giving sunrise as 6h 24min or 6:24 a.m., HST (Hawaii Standard Time).

Another example: at what time will the Sun be on the local meridian to an observer in Boston on October 31? Remember that the local meridian is the great circle from north to south passing through the zenith. The ET on October 31 is approximately +16 minutes, and Boston is situated at longitude 71^{o} W. The Apparent time when the Sun is on the meridian is 12 noon. Thus, Standard time = 12h 00min - (+16 min) = 11h 44min. This is the correct time at the Standard Meridian of 75^{o}W, but Boston is 4^{o} east of this, so the Sun crosses the Boston meridian 16 minutes before the 75th meridian. Hence, the answer is that the Sun crosses the meridian of Boston on October 31 at 11h 28min or 11:28 a.m., EST (Eastern Standard Time).

An interesting problem for the Hawaii student is to determine on what day and at what time the Sun passes directly overhead, i.e., through the local zenith. (Note that Hawaii is the only place in the United States where this can happen!) This requires the use of a table of the declination of the Sun for every day of the year and for the particular year in question, and such a table can be found in the American Ephemeris and Nautical Almanac (see REFERENCES). The declination of the Sun corresponds to latitude on the Earth, so that, when the declination of the Sun equals the latitude of the observer, the Sun will pass through the observer's zenith. Since the latitude of Hilo is 19^{o} 43', we need to look for the day on which the Sun has a declination of 19^{o} 43'. We then need to find the ET on that day and proceed to determine, as above, the HST at which that zenith crossing occurs. We like to call this a "shadowless" noon, for indeed, a flagpole has no shadow at that instant!

FURTHER READING

Some useful references:

1. **Stars Over Hawaii** by E. H. Bryan, Jr., was first published in 1955 by a company called BOOKS ABOUT HAWAII, but is long since out of print. A somewhat revised version was reprinted by The Petroglyph Press of Hilo, Hawaii, in 1977. Recently (September 2002) The Petroglyph Press (160 Kamehameha Avenue, Hilo, Hawaii 96720) has published a new version of **Stars Over Hawaii**, updated by Dr. Richard Crowe, Professor of Astronomy at UH-Hilo. This article, **Night and Day**, appears as an appendix in this new book.

Mr. Bryan, as a Curator of the Bernice Pauahi Bishop Museum in Honolulu was a fount of knowledge on many subjects related to Hawaii and the Pacific area. Although not a professional astronomer, he was one of the early active amateur astronomers in Hawaii and a frequent observer (back in the 1920's) of variable stars at the University of Hawaii's observatory in Kaimuki. In 1955 there was no source of basic information about the sky as seen from Hawaii and so Mr. Bryan filled a much felt need by writing this 48-page booklet about the Sun, stars, and planets as seen from Hawaii. Two sections of this booklet relate to the topics discussed in this paper. One has to do with how the Sun appears to move. Here he makes use of the polar plot to show the apparent path of the Sun in altitude and azimuth. In our Figures 7a to 10a we have borrowed this idea from him. In another section he discusses the situation when the Sun casts no shadow. Here is an interesting diagram showing the daily northward progression of the Sun over the islands during May and June. Anyone interested in Hawaiian astronomy will find this a useful addition to one's library.

2. **Astronomical Phenomena for the Year 2002**, or for succeeding years, is prepared annually by the Nautical Almanac Office, United States Naval Observatory, and jointly by Her Majesty's Nautical Almanac Office, Royal Greenwich Observatory. It is available from the U.S. Government Printing Office, Superintendent of Documents, Washington, D.C. 20402-9329. This approximately 80-page booklet contains much useful information for observational astronomy, including tables of sunrise, sunset, twilight, and day-by-day tabulations of the equation of time and the declination of the Sun. For exact work, the graph of the equation of time (Fig.6) in this paper is not sufficiently accurate and the tabulation in this booklet will be useful. This booklet is abstracted from a much more comprehensive compendium of astronomical data called The Astronomical Almanac, published annually by the Superintendent of Documents.

3. **Time in Astronomy**, Edmund Scientific Company, Barrington, N.J. 08007. Popular Optics Library No. 9054. Published in 1966 and reprinted in 1980; now possibly out of print. Written for the amateur astronomer, this booklet describes the various kinds of solar time and sidereal time, how to make time conversions, and how to locate astronomical objects in the sky. Many useful diagrams.

ABOUT THE AUTHORS

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George W. Bunton was for many years the Director of the Bishop Museum Planetarium and Science Center in Honolulu. He retired in 1981, and died on February 21, 1995. He was the true inspiration behind this paper and saw the original version completed just before his death.

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