Definition of a Cut for Multiplexing between Bolocam and High-Frequency Observations at the CSO
Version 3

Sunil Golwala

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Abstract

It is expected that the CSO’s observatory efficiency can be increased by weather-multiplexing high- and low-frequency observations. We define schemes for multiplexing between Bolocam low-frequency (150 and 275 GHz) and high-frequency (850 GHz) SHARC-II or heterodyne observations. The SHARC-II multiplexing scheme is a simple cut in $\tau_{225}$. The heterodyne multiplexing scheme makes use of the wide range of Bolocam mapping speeds obtained for a given atmospheric opacity to define a cut in the plane of $\tau_{225}$ vs. Bolocam mapping speed. With these multiplexing schemes, the observing speeds of the individual instruments are typically degraded by 20-30% relative to the speeds that would be achieved without multiplexing, so multiplexing is expected to improve observatory efficiency for high-priority scientific programs by about 50%.

1 Motivation and rough scheme

Due to the rarity and lack of predictability of low-opacity observing conditions at Mauna Kea (or elsewhere, for that matter), there is a great advantage to be gained in allocating observing time in a weather- or frequency-multiplexed fashion. In such a scheme, observing time is allocated directly to a low-frequency program ($\nu < 350$ GHz) that can make use of most reasonable observing conditions ($\tau_{225} < 0.15$, for example), with the stipulation that the telescope be handed over to a high-frequency program in low $\tau_{225}$ conditions. There should be significant improvement in observatory efficiency and science output: high-frequency programs will have access to all low-opacity periods, rather than only those low-opacity periods occurring during time allocated explicitly to such programs; and, periods of moderate to high opacity that are useless to high-frequency observations will be available to low-frequency observations rather than letting the telescope sit idle (or devoting such periods to relatively low-priority low-frequency backup programs).

Such a multiplexing scheme can only succeed if it is implemented for long enough stretches of time that many different weather conditions are averaged over. The extended (≃ 30-night) survey runs that are frequently allocated to Bolocam, the CSO’s 275 GHz and 150 GHz facility camera, meet this length criterion. This averaging ensures a predictable time split between high-frequency and low-frequency observations; the lost low-frequency observing time can be corrected for by simply allocating an equal additional fraction of time to the multiplexed program.

2 Mapping speed and opacity

The conventional scheme for such multiplexing uses a simple cut on $\tau_{225}$. However, an additional empirical fact of relevance is that the Bolocam mapping speed shows a large dispersion for any given
value of $\tau_{225}$. This suggests that some low-opacity conditions could be given up to high-frequency observations without significantly degrading Bolocam observations.

Mapping speed is defined empirically as

$$M_{\text{emp}} = \frac{\Omega_{\text{map}}}{\sigma_{\text{map}}^2 T_{\text{map}}}$$  \hspace{1cm} (1)$$

$\Omega_{\text{map}}$ is the size (solid angle) of an observed map, $\sigma_{\text{map}}$ is the rms noise achieved in the map (mJy) after beam-smoothing, and $T_{\text{map}}$ is the on-source time required to make the map. “On-source” time discards time spent in turnarounds, calibration, slewing, etc. in order to characterize pure instrument sensitivity. Mapping speeds are only quoted for data that has been subjected to sky-noise removal.

Mapping speed is a more useful figure of merit than NEFD for two reasons. First, because mapping speed takes into account the field of view of the instrument, it can be used directly to determine the time needed to cover some area to a desired depth. Second, mapping speeds add linearly, so the relative usefulness of conditions with different mapping speeds is intuitively clear (as opposed to NEFDs, which add as inverse squares). The latter point can be seen by considering the rms noise achieved in the optimal coadding of a set of observations using on-source time $t$ of some area $A$ with varying mapping speeds $M_k$. The variance is given by summing the inverse variances obtained in each observation:

$$\sigma_{\text{coadd}}^{-2} = \sum_k \sigma_k^{-2}$$  \hspace{1cm} (2)$$

$$= \sum_k M_k t$$  \hspace{1cm} (3)$$

$$= \frac{A}{t} \sigma_{\text{coadd}}^{-2} = \sum_k M_k$$  \hspace{1cm} (4)$$

So, mapping speeds add linearly in determining the inverse variance obtained on a given area.

Figure 1 shows scatter plots of $\tau_{225}$ vs. Bolocam mapping speed for the ensemble of 150 GHz and 275 GHz runs that have taken place to date. While it is clear that the median mapping speed does degrade as $\tau_{225}$ increases, it is also clear that low-opacity conditions do not necessarily provide good Bolocam mapping speeds. Broadly, the effect occurs because, at such low frequencies and for large focal plane arrays, atmospheric opacity is not a very important factor; rather the stability and “subtractability” of the atmosphere dominate. Anecdotally, we find the best Bolocam mapping speeds in conditions of low to moderate $\tau_{225}$ (0.05 to 0.1, perhaps as high as 0.15 when observing at 150 GHz) when $\tau_{225}$ is stable over many days. During such conditions, the exquisitely stable atmosphere can be removed so well that residual atmospheric noise is below Bolocam’s fundamental noise floor (limited by photon and detector noises).

It is observed that the high-frequency heterodyne observing speed depends only on $\tau_{225}$ (at least, no one has reported evidence to the contrary). Therefore, a multiplexing cut between Bolocam and high-frequency heterodyne observations in the plane of $\tau_{225}$ vs. Bolocam mapping speed may provide better observatory efficiency than a simple $\tau_{225}$ cut.

For multiplexing between Bolocam and SHARC-II, on the other hand, it is expected that there is nothing to be gained by a cut in the mapping speed–$\tau_{225}$ plane over a simple $\tau_{225}$ cut. It is expected (though no direct proof has been obtained yet) that, at a given value of $\tau_{225}$, the Bolocam and SHARC-II mapping speeds will degrade equally as conditions vary. This expectation can be justified on basic physics grounds. Atmospheric noise consists essentially of opacity fluctuations due to fluctuations in the precipitable water vapor (PWV) column depth along the line of site.
The same PWV fluctuations affect low-frequency and high-frequency observations. Thus, while the absolute fluctuation in opacity at two different wavelengths will be quite different because of the difference in opacity per unit PWV, the fractional fluctuations will be equal. It is the fractional fluctuations that matter for mapping speed, hence the expectation that Bolocam and SHARC-II mapping speeds will degrade proportionally. Given this expectation, a cut in mapping speed will always yield a zero-sum result. So, we only consider a $\tau_{225}$ cut for multiplexing between Bolocam and SHARC-II.

3 Defining a data set on which to base a cut

For Bolocam, each archived observation has a mapping speed and an on-source time associated with it. Obviously, we should create a two-dimensional histogram of the data in Figure 1. The mapping speed quantity itself is invariant under changes in the area mapped or time spent, so using mapping speed as an independent variable is sensible. In entering observations in the histogram, each entry should be weighted by its on-source time to account for variations in observation length. The resulting histogram will have a $z$-axis proportional to time.

Next, we divide the histogram by the total observing time of all entries in the histogram so the final histogram is “time fraction,” or, equivalently, the probability of obtaining a given mapping speed–$\tau_{225}$ combination.

Finally, following A. Kovacs’ suggestion, we correct the histogram for the fact that the $\tau_{225}$ conditions sampled by the data may not be typical. That is, if one projects the histogram along the mapping speed direction and makes a simple histogram as a function of $\tau_{225}$, the histogram will not look like the standard $\tau_{225}$ distribution. There are two reasons for this. First, there is...
simple sampling variance – the data set in hand is simply not big enough yet to have obtained a representative sample of $\tau_{225}$ values. Second, there is observing bias – at large values of $\tau_{225}$, the mapping speed is poor or the weather prevents the observer from opening the dome, so there tend to be fewer points at high $\tau_{225}$ than one expects from the distribution. There may be observing bias at low $\tau_{225}$ if a high-frequency multiplexing program was in effect at the time. These biases can be corrected for by simply comparing the observed $\tau_{225}$ distribution to the typical one, obtaining a scaling factor for each $\tau_{225}$ bin, and rescaling the 2D histogram bins by that factor. Once the 2D histogram has been rescaled, its projection onto the $\tau_{225}$ axis matches the typical distribution. The “typical” distribution we use is compiled from all winter data for calendar years 1997 through 2004.\footnote{Data obtained from CSO archive, summary distribution calculated by this author.} The corrected time fraction histograms are shown in Figure 2. The $\tau_{225}$ histograms before and after correction are shown in Figure 3. Note that the cumulative distribution does not reach 1 at high $\tau_{225}$ because some fraction of the time has $\tau_{225}$ above the upper limit of the $\tau_{225}$ axis.

There is one caveat: the correction assumes that, for a given $\tau_{225}$ bin, the distribution of mapping speeds in the uncorrected histogram is typical, so simply rescaling to the expected time fraction spent in that $\tau_{225}$ bin is the correct thing to do. This is valid when the occupancy of a $\tau_{225}$ bin differs from expectations because the actual $\tau_{225}$ distribution during the observations differed from the typical distribution. It is patently not valid when observing bias takes effect:

- For very low $\tau_{225}$ periods that may have been given to the high-frequency program, there may simply be no Bolocam data. Fortunately, in general there is some data in these conditions, enough to allow one to make the correction.\footnote{This is an important point for future observations – Bolocam must be allowed to sample some low $\tau_{225}$ conditions so that the low–$\tau_{225}$ mapping speed distribution can be measured accurately.}

- For high $\tau_{225}$ values, the conditions for which data is present in the histogram will tend to be the best conditions possible for that value of $\tau_{225}$ – in spite of the high $\tau_{225}$, it was possible to open the dome and observe. So, at high $\tau_{225}$ values, instead of just rescaling all the mapping speed bins by the same amount, the additional time fraction is conservatively put in the lowest mapping speed bin. This different treatment will in general not matter much because the contribution of high $\tau_{225}$ conditions to overall mapping speed is small.

To define a SHARC-II multiplexing cut, we require a similar $\tau_{225}$-mapping speed data set for SHARC-II.\footnote{Actually, we really only need the projection of the data set onto the $\tau_{225}$ axis since we have already decided to do a simple $\tau_{225}$ cut, but it is illuminating to carry along the full formalism and let the cut do the projection for us.} This has been provided by A. Kovacs. It also consists only of winter data. It has been corrected to the typical $\tau_{225}$ distribution in the same manner as above (again, using the 1997–2004 winter distribution). There is no low–$\tau_{225}$ observing bias issue, but the high–$\tau_{225}$ observing bias problem applies. Fortunately, there is a similar mitigating factor in that such observations would contribute negligibly to the SHARC-II overall mapping speed. The probability histogram and cumulative distribution analogous to Figures 2 and 3 are shown in Figure 4.

4 Defining the cut

4.1 Heterodyne cut

A cut is defined as a line in $\tau_{225}$ vs. $M$,

\[ \tau_{225} = m M + b \]  

\footnote{1}
Figure 2: Surface plots of time fraction vs. $\tau_{225}$ and mapping speed. Left: 150 GHz. Right: 275 GHz. Only “winter” data has been used (October through March), and the $\tau_{225}$ distribution has been corrected using the 1997-2004 “winter” distribution.

Figure 3: Cumulative distributions of $\tau_{225}$ for 150 GHz and 275 GHz data. The black line indicates the raw $\tau_{225}$ cumulative distribution, while the red line indicates the corrected distributions. The scaling factor needed to correct each $\tau_{225}$ bin is found by computing the differential version of each distribution and dividing the expected distribution by the actual one. The expected distribution is the 1997-2004 “winter” 1997-2001 distribution. The upper limit of the expected distribution is 0.92.
Figure 4: Left: Surface plot of time fraction vs. $\tau_{225}$ and mapping speed for SHARC-II. The data set has been rebinned in $\tau_{225}$ to match to more coarse Bolocam binning (because Bolocam samples a wider range of $\tau_{225}$ values, the statistics in each $\tau_{225}$ bin are smaller). Right: $\tau_{225}$ cumulative distribution before and after correction by typical $\tau_{225}$ distribution.

where $m$ is the line slope and $b$ its intercept with the $\tau_{225}$ axis. Bins below and above the line are devoted to high-frequency heterodyne observing and Bolocam observing, respectively.

We need a merit function to optimize the position of the cut. We first define a number of quantities that are a function of the cut position:

$$ f = \sum_{BC} f_i $$

$$ f_{corr} = f + \text{time fraction not included in histogram} $$

$$ \mathcal{H} = \frac{\sum_{hf} f_i \exp(-30 \tau_{225,i})^2}{\sum_{all} f_i \exp(-30 \tau_{225,i})^2} $$

$$ \mathcal{M} = \frac{\sum_{BC} f_i M_i}{\sum_{all} f_i M_i} $$

where $i$ is the histogram bin index (collapsed to a one-dimensional index) and $BC$, $hf$ and $all$ refer to bins that are above the cut line (are given to Bolocam), below the cut (are given to high-frequency heterodyne observing), and to all bins, respectively. $f_i$ is the histogram $z$ axis (time fraction), and $\tau_{225,i}$ and $M_i$ are the opacity and Bolocam mapping speed for the $i$th bin.

The first quantity $f$ is simply the fraction of time passing the cut. Because we consider only a limited range of $\tau_{225}$ in our histogram, the upper limit of $f$ is the same as the upper limit of the cumulative distribution in Figure 3. That upper limit is 0.92 for the $\tau_{225}$ distribution used to correct the raw histograms, so there is an unaccounted-for time fraction of 0.08. We define $f_{corr}$ to be time fraction after correction for this missing piece. The corrected time fraction should be

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4We somewhat nonintuitively use $M$ as the independent variable and $\tau_{225}$ as the dependent variable. The optimal cuts have shallow or no slope in this plane, which makes the slope computationally easy to deal with.
used when considering expectations for numbers of nights, though it does not figure into the cut optimization. The quantity $\mathcal{H}$ gives the speed at which heterodyne observations will be done using only the bins below the cut as compared to the speed obtained by using all the bins. Similarly, the quantity $\mathcal{M}$ is the speed at which Bolocam observations will be done by using only the bins above the cut as compared to the speed obtained by using all the bins. Note that both $\mathcal{H}$ and $\mathcal{M}$ are ratios of inverse variances, not rms depths.

Each instrument would like to maximize its own figure of merit, $\mathcal{H}$ or $\mathcal{M}$. Observatory efficiency is maximized by some tradeoff between the two. We define some possible joint figures of merit to optimize:

$$F_1 = \mathcal{M} \mathcal{H}$$
$$F_2 = \mathcal{M} + \mathcal{H}$$
$$F_3 = \sqrt{\mathcal{M}^2 + \mathcal{H}^2}$$

To find the optimal cut, we vary $m$ and $b$ over a wide range of values and find the values that maximize whichever figure of merit we prefer.

Scatter plots of $\mathcal{M}$ against $\mathcal{H}$ are shown in Figure 5. Figure 5 also shows lines of constant $F_1$, $F_2$, and $F_3$. The $F_1$ figure of merit has the most power for selecting an optimal cut because the shape of its curve of constant value is most unlike the locus of candidate cut points in Figure 5. Plots of the combined figures of merit against the Bolocam time fraction $f$ are shown in Figure 6.

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Footnotes:

5Scaling of observing speed as $[\exp(-30 \tau_{225})]^2$ provided by T. Phillips.
Figure 6: Scatter plot of joint figures of merit $F_1$, $F_2$, and $F_3$ vs. (uncorrected) time fraction given to Bolocam $f$. Again, there is one point for each candidate cut. The red squares indicate the cuts that provided the combined figure of merit $F_1$ within 10% of its maximum value. Left: 150 GHz. Right: 275 GHz.
Figure 7: Scatter plot of Bolocam observing speed figure of merit $\mathcal{M}$ vs. SHARC-II observing speed figure of merit $S$. Each point provides the figure of merit for one candidate cut. The three solid curves indicate lines of constant $S_1$ (hyperbolic), $S_2$ (linear), and $S_3$ (circular). The red squares indicate the cuts that provided the combined figure of merit $S_1$ within 10% of its maximum value. Left: 150 GHz. Right: 275 GHz.

4.2 SHARC-II cut

The SHARC-II multiplexing cut will be a simple $\tau_{225}$ cut. The cut is defined as a straight horizontal line in the $\tau_{225}$ vs. Bolocam mapping speed. The figures of merit $f$ and $\mathcal{M}$ are calculated as before. The SHARC-II figure of merit will be similar to the Bolocam figure of merit, except that it will obviously be calculated in the plane of $\tau_{225}$ vs. SHARC-II mapping speed:

\[
S = \frac{\sum_{SH2} f_i S_i M_i S_i}{\sum_{all} f_i S_i M_i S_i}
\]  

(13)

$SH2$ refers to the bins that are below the $\tau_{225}$ cut line (and thus used for SHARC-II observing). $f_i^S$ and $M_i^S$ are defined in the $\tau_{225}$ vs. SHARC-II mapping speed plane in analogy to their definitions for Bolocam. Since we are only considering $\tau_{225}$ cuts, there is no ambiguity in how a cut translates between the Bolocam and SHARC-II mapping speed–$\tau_{225}$ planes; we project out that dimension when calculating $\mathcal{M}$ and $S$.

We define similar figures of merit

\[
S_1 = \mathcal{M} S
\]

(14)

\[
S_2 = \mathcal{M} + S
\]

(15)

\[
S_3 = \sqrt{\mathcal{M}^2 + S^2}
\]

(16)

We show a scatter plot of $\mathcal{M}$ vs. $S$ in Figure 7 and plots of the three figures of merit as a function of $f$ in Figure 8.
Figure 8: Scatter plot of joint figures of merit $S_1$, $S_2$, and $S_3$ vs. (uncorrected) time fraction given to Bolocam $f$. Again, there is one point for each candidate cut. The red squares indicate the cuts that provided the combined figure of merit $S_1$ within 10% of its maximum value. Left: 150 GHz. Right: 275 GHz.
Figure 9: Contour plots of time fraction vs. $\tau_{225}$ and mapping speed (same information as Figure 2), with candidate cuts displayed. The tables in the plots indicate the (uncorrected) fraction of time, $f$, and figures of merit as a function of cut parameters. For reference, the normalization of $M$, which is the Bolocam mapping speed obtained when no cut is applied, is provided ($\langle M \rangle$). Left: 150 GHz. Right: 275 GHz.

5 Cut definition results

The effect of the cuts is summarized in scatter plots shown in Figures 5, 6, 7, and 8, which have one point for each cut candidate pair ($m, b$). (For the SHARC-II multiplexing cut, only $m = 0$ cuts are considered.) In all plots, those candidate cuts within 10% of the maximum value of $F_1$ or $S_1$ are indicated with red squares. These cuts are less clearly optimal with respect to the other figures of merit for reasons given above. Since $F_1$ and $S_1$ most clearly pick optimal cuts, and the other figures of merit have less discriminating power, we rely on the $F_1$ and $S_1$ figures of merit.

The same sets of best candidate cuts are shown in Figures 9 and 10. Tables of cut parameters, $f$, and figures of merit are shown in the plots. It should of course be realized that the mapping speed distributions are not yet fully sampled, so small variations in figures of merit should not be assigned too much significance.

It is clear that cuts can be defined that have noticeable but not egregious impact on both kinds of observations: losses of 20-30% in observing speed are possible, which correspond to losses of 10-17% in achieved sensitivity.\(^6\) For heterodyne multiplexing, there are multiple cuts that yield unnoticeably different figures of merit, so it is somewhat arbitrary which one of these to pick. Obviously, a cut that yields similar Bolocam and heterodyne figures of merit is most equitable. The shallowest possible cut is preferred because the lowest $\tau_{225}$ conditions are obviously preferred for high-frequency observing. A cut with zero slope (a simple $\tau_{225}$ cut) has the significant advantage of being much simpler to implement because it requires no mapping speed measurement.\(^7\)

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\(^6\)How to adjust allocations to deal with these losses is discussed in Section 6.

\(^7\)But, if it has smaller Bolocam figure of merit $M$, such a cut will require a larger correction to time allocations.
Figure 10: Contour plots of time fraction vs. $\tau_{225}$ and mapping speed (same information as Figure 4), with candidate cuts displayed. The top two figures are for Bolocam, the bottom two for SHARC-II. The tables in the plots indicate the (uncorrected) fraction of time, $f$, and figures of merit as a function of cut parameters. For reference, the normalizations for $M$ and $S$, which are the Bolocam and SHARC-II mapping speeds obtained when no cut is applied, are provided ($\langle M \rangle$ and $\langle S \rangle$). Left: 150 GHz. Right: 275 GHz.
The proposed cuts are given in Table 1.

The above cuts improve observatory efficiency simply by the sum of the two individual figures of merit, either $M + H$ or $M + S$. That is, rather than obtaining observing speed $M = 1$ or $H = 1$ separately without multiplexing, we achieve $M$ and $H$ simultaneously. Gains of about 50% in observatory efficiency are obtained. Taken literally, this is an overestimate because most high-frequency observers have a low-frequency backup program. However, exercising some prejudices about scientific priority, the explicitly multiplexed program allows the CSO to simultaneously and effectively pursue two high-priority science goals – high-frequency line searches and continuum mapping and low-frequency continuum surveys – rather than using the high-opacity time for relatively low-priority low-frequency backup programs.

6 Using the cuts to define a multiplexing strategy

We would like to use the above cuts to define a strategy for multiplexing. Many different factors enter into such a strategy – not just the placement of the cut, but how to correct time allocations, under what conditions to not implement the multiplexing program, how to compensate observers whose allocated time has been given up to high-frequency observing, etc. We consider these questions in this section.

6.1 Extending allocations for long survey runs

A multiplexing program can be most easily and successfully implemented during long Bolocam survey runs because a wide range of weather and opacity conditions will be sampled. The most straightforward and efficient way to request and allocate the time is as follows:

- A proposer wants to reach a desired depth $\sigma_p$. The proposer can calculate the time needed based on the average mapping speed in the absence of multiplexing, $\langle M \rangle$, as provided in Table 1. The time request takes into account the typical $\tau_{225}$ distribution, as should be done for long survey runs. Let $N_p$ be the number of nights proposed.

- The TAC will allocate a number $N_a$ nights to the survey, realizing that the depth will degrade as $\sigma_a/\sigma_p = \sqrt{N_p/N_a}$. This allocation should be made without respect to multiplexing.

- Assuming multiplexing will be implemented for the entire run, increase the allocation by a factor $1/M$ to give a total number of allocated real nights $\tilde{N}_a = N_a/M$.

In the $\tilde{N}_a$ nights, with multiplexing, Bolocam is expected to reach the same depth it would have reached in $N_a$ nights without multiplexing. Note that $f$ is irrelevant; the allocation needs to be corrected not by a simple time fraction, but by the degradation in mapping speed. Correcting by $f$ would be incorrect because the nights that are given to the high-frequency program are not random, but instead are taken from a specific region in the mapping speed distribution.

Consider an example involving multiplexing with the heterodyne program. Suppose there is a 275 GHz survey request for 30 nights and suppose that 30 nights are allocated by the TAC. We have $N_p = 30$ and $N_a = 30$. The “equitable” cut has $M = 0.77$, so the allocation should be increased to $\tilde{N}_a = N_a/M = 30/0.77 = 1.30 \times 30 = 39$ nights.

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for multiplexing; see Section 6.
Table 1: Proposed cuts based on algorithm described in text. Since many different cuts give approximately the same combined figure of merit, we indicate for each case the cut that gives the most equitable degradation of observing speed (“equitable”) and the cut that gives a simple $\tau_{225}$ cut (“flat”). The values of $f$, $f_{corr}$, $M$, $H$, and $S$ (as defined in the text) are given. (Recall that $f_{corr} = f + 0.08$.) The sums $M + H$ and $M + S$ give the improvement in observatory efficiency over unmultiplexed observing. The average mapping speeds in the absence of a cut for Bolocam and SHARC-II are provided ($\langle M \rangle$ and $\langle M^S \rangle$). As will be explained in Section 6, we also list $f_{obs}$, the fraction of the time with $\tau_{225} < 0.15$, which is what we consider “observable” time, and $f_{corr,obs}$, the expected fraction of “observable” time to be used by Bolocam.
6.2 Calculating expectations for the multiplexed night distribution

During an observing run, we expect that a fraction \( f_{\text{corr}} \) of nights will be given to Bolocam and a fraction \( 1 - f_{\text{corr}} \) will be given to high-frequency observations. Observers will certainly compare the number of nights they are actually getting to these expectations. There will no doubt be fluctuations due to the vagaries of the weather. Given such variations, how do we determine what the expected distribution of nights should be?

We are worried about times when \( f_{\text{obs}} \), the fraction of “observable” nights (which we somewhat arbitrarily define as nights with \( \tau_{225} < 0.15 \)), deviates severely from its expected value. For example, in the above example, where 39 total nights are allocated, including multiplexing, we expect that Bolocam will make use of a fraction \( f_{\text{corr}} \) of the time, so \( 0.80 \times 39 = 31 \) nights. The remaining fraction \( 1 - f_{\text{corr}} = 0.20 \), or 8 nights, would be used by the high-frequency program. We expect that of the 39 nights, a fraction \( f_{\text{obs}} = 0.74 \) should have \( \tau_{225} < 0.15 \), or 29 nights. Suppose, however, that there was a bad stretch of weather so that there were actually only 15 observable nights. (This has been known to happen!). The 24 other nights are nominally Bolocam nights, but clearly they are unusable. Obviously, it would be unfair to maintain the requirement that the high-frequency program get 8 nights when such a large fraction of the nights were unusable.

A reasonable method for correcting for such difficulties is to reference the expectations to the actual number of observable nights. Specifically: Bolocam expects to get a fraction \( f_{\text{corr}} \) of all nights. A fraction \( f_{\text{obs}} \) of nights are observable. All the unobservable nights are Bolocam nights, so it is expected that a fraction \( f_{\text{corr}} - (1 - f_{\text{obs}}) \) of all nights will be observable and given to Bolocam. The fraction of observable nights that will be given to Bolocam is then

\[
f_{\text{corr},\text{obs}} = \frac{f_{\text{corr}} - (1 - f_{\text{obs}})}{f_{\text{obs}}} \tag{17}
\]

That is, if we get \( N_{\text{obs}} \) observable nights, then we expect that Bolocam will get a fraction \( f_{\text{corr},\text{obs}} \) of these \( N_{\text{obs}} \) nights and the high-frequency program will get a fraction \( (1 - f_{\text{corr},\text{obs}}) \) of these nights. In the above example, where \( N_{\text{obs}} = 15 \), the expectations would be \( f_{\text{corr},\text{obs}} \times N_{\text{obs}} = 0.73 \times 15 = 11 \) nights and the expected number of high-frequency nights would be 4. These expectations based on the actual \( N_{\text{obs}} \) should be the reference points for observers in deciding whether the night distribution being obtained is reasonable.

6.3 Deciding what is an “unreasonable” deviation from expectations

Given the above weather-corrected expectations for the night distribution, how do we determine whether the actual night distribution is within a “reasonable” range of the weather-corrected expectations? Clearly, we would not want to force the night distribution to match expectations exactly on any given day because this could result in nonsensical assignment of nights – e.g., allocation of a \( \tau_{225} = 0.1 \) night to high-frequency observing just because the number of high-frequency nights is one night short of expectations. There will always be fluctuations about the expected distribution, so how do we define what is an “unreasonable” deviation that needs to be corrected by reassignment of nights without regard to the cut?

“Reasonable” can be defined based on the expected probability distribution of the number of nights. With \( N_{\text{obs}} \) and \( f_{\text{corr},\text{obs}} \) as defined above, the actual number of low-frequency nights, \( N_{\text{l}} \), is a random variable following a binomial distribution parameterized by \( N_{\text{obs}} \) and \( f_{\text{corr},\text{obs}} \):

\[
P(N_{\text{l}}, N_{\text{obs}}, f_{\text{corr},\text{obs}}) = \frac{N_{\text{obs}}!}{N_{\text{l}}!(N_{\text{obs}} - N_{\text{l}})!} f_{\text{corr},\text{obs}}^{N_{\text{l}}}(1 - f_{\text{corr},\text{obs}})^{N_{\text{obs}} - N_{\text{l}}} \tag{18}
\]
Few would argue that this range is unreasonable, so we take it as our “reasonably likely” range. The number of high-frequency nights follows a complementary distribution, the binomial distribution becomes Gaussian and this 90% most likely range is the expected value of $N_{lf}$ when the randomly distributed variable is discretized. A short IDL routine can provide the values in this region equals or exceeds 90%. This method for defining such a region (called the Neyman construction) is statistically rigorous, though it can lead to overcoverage (obtaining a region that has probability larger than 90%) when the randomly distributed variable is discretized. A short IDL routine can provide the $\alpha$% most likely range for any value of $\alpha$, $N_{obs}$, and $f_{corr,obs}$.

The definition of “reasonably likely” is now the arbitrary criterion. We arbitrarily choose the 90% most likely range as the suitable one. In the limit of large $N_{obs}$ and $\sqrt{f_{corr,obs} N_{obs}} \ll f_{obs}$, the binomial distribution becomes Gaussian and this 90% most likely range is the ±1.64σ range. Few would argue that this range is unreasonable, so we take it as our “reasonably likely” range.

To illustrate this correction method, consider the example $N_{obs} = 15$ and $f_{corr,obs} = 0.73$. The expected value of $N_{lf}$ is $\langle N_{lf} \rangle = 11.0$. The 50% most likely range is $N_{lf} \in [10, 12]$ and the 90% most likely range is $N_{lf} \in [8, 13]$. If the actual value of $N_{lf}$ obtained is outside the 90% most likely range, then a correction is enforced. For example, if $N_{lf}$ exceeds 13, then nights should be given to the high-frequency program to bring $N_{lf}$ back into the 90% most likely range. Smaller deviations from $\langle N_{lf} \rangle = 11.0$ should be ignored.

One final note: if a correction is warranted, the observer receiving the correction night should of course use judgement in deciding whether to accept it. If a particularly anomalous stretch of weather results in $\tau_{225} < 0.15$ for only 15 nights but no nights with $\tau_{225} < 0.1$, presumably the high-frequency observer would have little use for a $\tau_{225} > 0.1$ night and would choose to not accept the correction night. Similarly, a low-frequency observer may choose to give up a correction night if the observer finds the Bolocam mapping speed is horrific. These choices are up to the observer being given the correction night.

### 6.4 Short runs

The situation is more difficult for short runs (less than 1-2 weeks in length). The issue is that the weather is coherent over short timescales so the law of averages does not apply. Requests for time and time allocations should not correct for the “observable” time fraction because it is likely that either none or all of the time will be observable. When time is allocated, there should similarly be no correction for the reduction in effective mapping speed due to multiplexing. For example, if a proposal requires 4 nights of “observable” conditions to reach its desired depth, it should only be allocated 4 nights.

To be fair, though, there should be both restrictions on the total number of nights that can be lost and compensation for lost nights. Given the cost and inconvenience of travel the “long survey” 90% confidence region caps are too liberal to be used to restrict the night distribution for short runs. For example, for a 4 night run with only 3 nights observable, the 90% most likely region is $[1, 3]$, meaning that it is possible for the low-frequency observer to get as little as 1 night out of the 3 observable nights. For a 2 night run with 1 observable night, the 90% most likely region is $[0, 1]$, meaning the low-frequency observer could get 0 nights. (Remember, the 90% most likely region is

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8The “90% most likely” values for $N_{lf}$ are found by sorting all possible values of $N_{lf}$ by their $P$ value (maximum $P$ first) and then adding values of $N_{lf}$ to the 90% most likely region until the total probability of obtaining any of the values in this region equals or exceeds 90%. This method for defining such a region (called the Neyman construction) is statistically rigorous, though it can lead to overcoverage (obtaining a region that has probability larger than 90%) when the randomly distributed variable is discretized. A short IDL routine can provide the $\alpha$% most likely range for any value of $\alpha$, $N_{obs}$, and $f_{corr,obs}$.

9Square brackets indicate the range is inclusive of the endpoints.

10These ranges actually overcover – they have slightly larger than 50% and 90% probability, respectively – because $N_{lf}$ is discretized as explained in a previous footnote.
overcovered because of quantization of nights.). How to modify this is somewhat arbitrary. One reasonable policy is that there should be a floor of 50% of the expected number of nights, rounded upward to the nearest integral number of nights. For example, for a 2 night run at 275 GHz, the expected number of observable nights is $0.74 \times 2 = 1.5$ and the expected number of observable nights that would go to Bolocam is $0.74 \times 0.73 \times 2 = 1.08$, so the floor would be 0.54, which rounds to 1. That is, the scheduled observer should get at least 1 observable night (assuming there are any observable nights to give). For a 5 night run at 275 GHz, the floor is $0.74 \times 0.73 \times 0.5 \times 5 = 1.35$, which rounds to 2 nights.

The transition between this hard floor and the 90% most likely region will depend on $f_{\text{obs}}$ and $f_{\text{corr,obs}}$, but occurs around $N_a = 9$ nights. The simplest way to transition between the hard floor and the 90% most likely region is to simply take whichever is largest. The hard floor will be larger at small $N_a$ and the lower edge of the 90% most likely regions will be larger at large $N_a$.

Finally, for such short allocations, if the observer gives up nights to the high-frequency program, then the observer should be automatically compensated for the lost nights in a later semester without having to repropose.

### 6.5 Manpower Issues

Another important aspect is how to distribute manpower for multiplexing arrangements. For long runs, this becomes especially difficult: since $\mathcal{M} \approx 0.7$, the runs have to be extended by a factor $1/0.7 \approx 1.4$ to compensate for the lost time. In the past, the manpower arrangement has been that the low-frequency program must schedule 2 people for all nights, with the high-frequency observer taking the place of one of the low-frequency observers on nights given to the high-frequency program. Since it is not known ahead of time which nights will be high-frequency nights, there must be 2 low-frequency observers present at HP for the entire extended run. The low-frequency program must therefore provide 40% more manpower than it would have in the absence of multiplexing, with no improvement in depth over unmultiplexed observations. Moreover, there has to be a high-frequency observer resident in Hawaii for the entire run anyways, so there is substantial operational inefficiency.

One obvious way to correct this is for the low-frequency and high-frequency programs to pool observers, with each program providing a fraction of the manpower in proportion to its time fraction. This is nominally equitable, but it results in operational difficulties because usually observers from a given program are not trained to observe at the other frequency.

Another solution is to split the manpower equally between the low-frequency and high-frequency programs so each program has an observer at the summit on any given night. The high-frequency observers might view this as unequitable, as they will only use of order 30% of the nights. But, when viewed in terms of observing speed, it is entirely equitable – to obtain the same depth of observation without multiplexing, the high-frequency program would have had to provide 2 observers for a run of length a fraction $\mathcal{H}$ of the actual run length. For example, consider our 30 night run that is extended to 39 nights. The high-frequency figure of merit $\mathcal{H}$ or $\mathcal{S}$ is approximately 0.75. This means that the high-frequency program achieves in the 39 multiplexed nights the same depth it would have achieved in $0.75 \times 39 = 29$ unmultiplexed nights. The high-frequency program would have had to provide 2 observers for those 29 nights. So, having to provide 1 observer for 39 nights is a substantial improvement. This seems like the most sensible solution.

For short runs, because the manpower needs are not extreme, reverting to the current scheme, with a high-frequency observer resident in Hawaii but not required to be at the summit every night, would probably be fine. This also compensates the high-frequency observer to some extent for the restrictions placed on the number of high-frequency nights that can be taken during short runs.